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Transmission of Sound through Air at Low Pressure

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THE famous elementary physics demonstration of the electric bell ringing in a bell jar from which the air is gradually pumped out, and the corresponding decrease in the intensity of the sound that is heard, is familiar to all. We tell our students that this experiment proves that a material medium is necessary for the transmission of sound—one of those glib statements which make one wonder whether it is ever possible to tell the truth about anything in an elementary physics course. Actually the experiment demonstrates the impedance mismatch between transducer and surrounding medium, but an understanding of this explanation demands a knowledge of the theory of wave propagation.

This paper is limited to a survey of the attenuation of sound in air as a function of pressure or density. For the purpose of unification and simplification we shall treat the problem in terms of relaxation dissipation.

Wave Propagation with Relaxation Dissipation

The usual treatment of relaxation dissipation in compressional-wave propagation leads to a complex wave velocity which, when substituted into the ordinary harmonic wave solution of the standard wave equation, provides for the appearance of the dissipative term. It seems worth while to present here a somewhat different point of view, which removes some of the mystery associated with the complex velocity and also emphasizes the fact that the equation we have to deal with in the description of attenuation is not the standard wave equation anyway.

For simplicity we shall confine our discussion to plane compressional waves in one dimension. If we denote the displacement of the elastic medium in the x direction by ξ, the excess pressure by p_e and the mean density by ρ_0, the usual hydrodynamic equation of motion for small departures from equilibrium is

$$\rho_0(\partial^2\xi/\partial t^2) = -(\partial p_e/\partial x). \tag{1}$$

In the common method of deriving the wave equation, Eq. (1) is supplemented by Hooke's law in the form

$$p_e = B\rho_e/\rho_0 = -B(\partial\xi/\partial x), \tag{2}$$

where ρ_e is the excess density and B the bulk modulus. Elimination of p_e between Eqs. (1) and (2) yields the well-known equation

$$\partial^2\xi/\partial t^2 = c^2(\partial^2\xi/\partial x^2), \tag{3}$$

where the wave velocity is

$$c = (B/\rho_0)^{\frac{1}{2}}. \tag{4}$$

Now Eq. (2) is a static-equilibrium equation which ignores the dynamical mechanism by which the density is changed through the agency of the excess pressure. To take account of the latter we must replace this equation by the more accurate pressure *relaxation* equation

$$p_e = B\rho_e/\rho_0 + R\dot{\rho}_e, \tag{5}$$

where partial differentiation with respect to

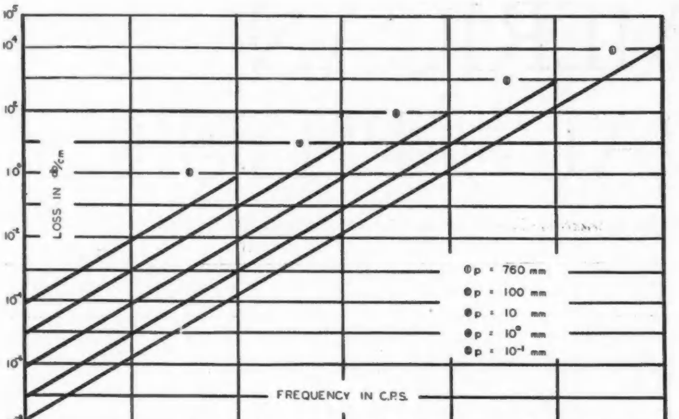

FIG. 1. Classical approximation: $\alpha\lambda<1$; 2α(db/cm) as a function of frequency for various pressures.

time is indicated by the use of the dot. The quantity R may be called the relaxation constant. The corresponding relaxation time is $\rho_0 R/B$. This is the time taken by the density to attain to the fraction $1-1/e$ of the equilibrium value corresponding to a given imposed excess pressure. If we employ $\rho_e=-\rho_0(\partial\xi/\partial x)$, we can rewrite Eq. (5) in the form

$$p_e=-B\frac{\partial\xi}{\partial x}-R\rho_0\frac{\partial\dot{\xi}}{\partial x}. \tag{6}$$

Elimination of p_e between Eqs. (1) and (6) now leads to the equation

$$\frac{\partial^2\xi}{\partial t^2}=(B/\rho_0)\frac{\partial^2\xi}{\partial x^2}+R\frac{\partial^2\dot{\xi}}{\partial x^2}, \tag{7}$$

which is formally equivalent to the Stokes equation for the transmission of a plane compressional wave through a viscous medium[1] if R is treated as proportional to the viscosity coefficient μ. Indeed, in the standard treatment of viscosity dissipation,

$$R=4\mu/3\rho_0. \tag{8}$$

However, for the moment let us proceed without too great specialization. It will be sufficient to consider a harmonic particular solution of Eq. (7) in the form

$$\xi=\exp(-\alpha x)\cos(\omega t-kx). \tag{9}$$

[1] See Lord Rayleigh, *Theory of sound*, (Dover Publications, 1945), sec. 346, Eq. (4).

Substitution shows that this is a solution of Eq. (7) provided

$$2\alpha k=\frac{\omega^3\tau\rho_0}{B(1+\omega^2\tau^2)}, \tag{10}$$

and

$$k^2-\alpha^2=\frac{\omega^2\rho_0}{B(1+\omega^2\tau^2)}, \tag{11}$$

where

$$\tau=\rho_0 R/B \tag{12}$$

is the relaxation time. The elimination of k between Eqs. (10) and (11) yields for α, which is the amplitude-attenuation coefficient, the biquadratic equation

$$\alpha^4+\frac{\omega^2\rho_0}{B(1+\omega^2\tau^2)}\alpha^2-\frac{\omega^6\tau^2\rho_0^2}{4B^2(1+\omega^2\tau^2)^2}=0. \tag{13}$$

The real solution is

$$\alpha=\left(\frac{1}{2B/\rho_0}\right)^{\frac{1}{2}}\omega\left[\frac{(1+\omega^2\tau^2)^{\frac{1}{2}}-1}{(1+\omega^2\tau^2)}\right]^{\frac{1}{2}}. \tag{14}$$

This is in rather awkward form. If we solve for k from Eq. (10) we get

$$k=\frac{\omega^2\tau\{(1+\omega^2\tau^2)[(1+\omega^2\tau^2)^{\frac{1}{2}}-1]\}^{-\frac{1}{2}}}{(2B/\rho_0)^{\frac{1}{2}}}. \tag{15}$$

But this is ω/V, where V is the phase velocity of the wave. Hence

$$V=\frac{(2B/\rho_0)^{\frac{1}{2}}}{\omega\tau}\{(1+\omega^2\tau^2)[(1+\omega^2\tau^2)^{\frac{1}{2}}-1]\}^{\frac{1}{2}}. \tag{16}$$

If now we express α in terms of V, the result is

$$\alpha = \frac{\omega^2 \tau V}{(2B/\rho_0)(1+\omega^2\tau^2)}. \tag{17}$$

Since V depends on ω—that is, there is dispersion—Eq. (17) does not exhibit directly the dependence of α on the frequency. The ultimate approximate form will depend on the magnitude of τ.

Viscosity and Heat-Conduction Dissipation

Viscosity dissipation can be treated by the method of the preceding section. Here, from Eqs. (8) and (12),

$$\tau = 4\mu/3B. \tag{18}$$

By simple kinetic theory, μ is independent of the pressure. On the other hand, the adiabatic bulk modulus is γp, where p is the pressure and γ the ratio of the specific heat at constant pressure to that at constant volume. Since for air at room temperature μ is 1.8×10^{-4} dyne sec/cm^2, it follows that for pressures above 1 dyne/cm^2 and even considerably below, the value of τ in seconds is very small compared with unity. For not too high frequencies we therefore have $\omega\tau \ll 1$. Under these circumstances Eq. (17) reduces to the conventional form

$$\alpha = 2\mu\omega^2/3\rho_0 c^3, \tag{19}$$

where

$$c = (B/\rho_0)^{\frac{1}{2}}. \tag{20}$$

The effect of heat conduction also is, to a first approximation, a relaxation effect. We shall not discuss it here.[2] It is well known that it leads, for sufficiently low values of thermal conductivity and not too high frequencies, to an absorption coefficient α of the form

$$\alpha = \frac{\omega^2}{2\rho_0 c^3}\left(\frac{\gamma-1}{\gamma}\right)\frac{\eta}{c_v}, \tag{21}$$

where η is the thermal conductivity and c_v the specific heat at constant volume. The corresponding value of the relaxation time becomes

$$\tau = \left(\frac{\gamma-1}{\gamma}\right)\frac{\eta}{c_v B}. \tag{22}$$

[2] See Lord Rayleigh, reference 1, sec. 349.

It is of interest to observe that for air at normal atmospheric pressure and room temperature, the value of τ calculated from either Eq. (18) or Eq. (22) is of the order of 10^{-10} sec.

We now introduce the standard kinetic theory results,

$$\eta = 2c_v\mu, \tag{23}$$

$$\mu = 0.3\rho_0 v_m l, \tag{24}$$

where l is the mean free path and v_m the root-mean-square molecular velocity. Using the formulas $v_m = (3p/\rho)^{\frac{1}{2}}$ and $c = (\gamma p/\rho)^{\frac{1}{2}}$, the latter being sufficiently accurate for an approximate calculation, we get for the *intensity* absorption coefficient, taking into account both viscosity and heat conduction,

$$2\alpha = \frac{23.6\nu^2 l}{c^2}(3/\gamma)^{\frac{1}{2}}\left(\frac{2}{3}+\frac{\gamma-1}{\gamma}\right). \tag{25}$$

If we specialize at once to air with $\gamma = 1.41$ and $c = 3.44\times10^4$ cm/sec at 20°C, this becomes in reciprocal centimeters

$$2\alpha = 2.77\nu^2 l\times10^{-8}, \tag{26}$$

if ν is frequency in cycles per second and l is the mean free path in centimeters. For the mean free path we use the standard Clausius formula,

$$l = 3m/4\pi D^2\rho_0, \tag{27}$$

where m is the mass and D the diameter of the molecule. Using $D = 3.1\times10^{-8}$ cm as the diameter of the nitrogen molecule (that of oxygen is about 3×10^{-8} cm) and $m = 46.5\times10^{-24}$ g, we obtain for the mean free path, if ρ_0 is in grams

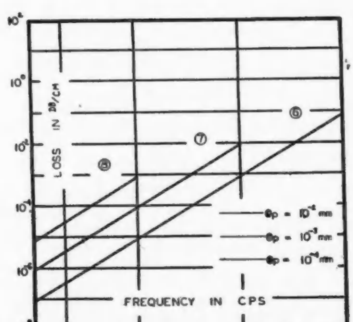

FIG. 2. Classical approximation: $\alpha\lambda < 1$; 2α (db/cm) as a function of frequency for various pressures.

per cubic centimeter,

$$l = 1.15 \times 10^{-8}/\rho_0, \tag{28}$$

and 2α becomes, in reciprocal centimeters,

$$2\alpha = 3.19 \times 10^{-16}\nu^2/\rho_0. \tag{29}$$

In more conventional units the transmission loss in decibels per centimeter is

$$2\alpha = 13.9 \times 10^{-16}\nu^2/\rho_0. \tag{30}$$

It must be recalled from (17) coupled with $\omega\tau \ll 1$ that the limit on the validity of Eq. (30) is given by $\alpha\lambda < 1$ or

$$5.5 \times 10^{-12}\nu/\rho_0 < 1. \tag{31}$$

In Fig. 1 we present a logarithmic graph of Eq. (30) for various air pressures, from 760 to 10^{-1} mm of mercury. Figure 2 extends this to pressures from 10^{-2} to 10^{-4} mm of mercury. The frequency limits to the validity are indicated by the length of the line in each case. Thus at atmospheric pressure, Eq. (30) is approximately valid for $\nu < 10^8$ c/sec. Obviously the upper frequency limit for validity decreases in the same proportion as the pressure. Except for sound of frequency less than 1 c/sec, Eq. (30) has no validity for pressures lower than 10^{-5} mm of mercury.

It is interesting to note that the experimental values of absorption in dry air at atmospheric pressure and 26.5°C quoted by Sivian[3] correspond to

$$2\alpha = 19.7 \times 10^{-16}\nu^2/\rho_0 \text{ (db/cm)},$$

with a good frequency-squared variation from 20 to 200 kc/sec. The increase over the theoretical value given by Eq. (30) is not one order of magnitude and is not sufficient to show up on the scale used in Fig. 1.

Calculations of Tsien and Schamberg

Recently Tsien and Schamberg[4] have employed the second-order approximation to the viscous stresses and the heat flow given in the more recent kinetic theory considerations of Chapman and Cowling[5] to calculate the absorption of plane sound waves in rarefied mediums. They obtain an expression for 2α in the form of a power series in ν^2. When the values for dry air at 20°C are inserted the series becomes

$$2\alpha = 9.04 \times 10^{-10}(\nu^2/p)(1 - 9.15 \times 10^{-13}\nu^2/p^2 + 1.34 \times 10^{-24}\nu^4/p^4 + \cdots). \tag{32}$$

Here p is the pressure in millimeters of mercury. Reference 4 shows that the convergence is assured only if the Reynolds number

$$R' = 1.64 \times 10^6 p/\nu > 1, \tag{33}$$

or

$$\nu/p < 1.64 \times 10^6.$$

In terms of density ρ_0, this condition becomes

$$0.97 \times 10^{-12}\nu/\rho_0 < 1. \tag{34}$$

For given ρ_0 this provides a somewhat greater range of frequency validity than the simple classical theory as exemplified in Eq. (31), though the order of magnitude is the same. As a matter of fact, the contribution of the correction terms in Eq. (32) in the region of validity is very small, too small indeed to show up in Figs. 1 and 2.

Classical Approximation for Very High Frequency

The relatively small magnitude of the correction introduced by the considerations of Tsien and Schamberg encourages us to examine once again the classical expressions for α for ultra high frequencies at ordinary pressures or ordinary frequencies at very low pressures. It is indeed too much to expect that these expressions will have more than order-of-magnitude significance, and further work along the lines of the theory of nonuniform gases is definitely indicated. If we revert to Eq. (14) we see that for $\omega\tau \gg 1$, α assumes the very simple form

$$\alpha = 2\pi\nu(2\omega c^2\tau)^{-\frac{1}{2}}, \tag{35}$$

where for air, using the data already given

$$\omega\tau = 1.23 \times 10^{-12}\nu/\rho_0. \tag{36}$$

[3] L. J. Sivian, *J. Acous. Soc. Am.* **19**, 914 (1947).

[4] H. S. Tsien and R. Schamberg, *J. Acous. Soc. Am.* **18**, 334 (1946). As pointed out by C. S. Wang Chang and G. E. Uhlenbeck, there exists an error in the numerical values taken by Tsien and Schamberg from Chapman and Cowling (reference 5). This led to incorrect values for b_3 and b_5 in Table I of Tsien and Schamberg's article. When corrected these become $b_3 = -2.29$, $b_5 = +9.05$, and these values are used in the present review to calculate 2α in Eq. (32).

[5] S. Chapman and T. G. Cowling, *The mathematical theory of non-uniform gases* (Cambridge Univ. Press, 1939).

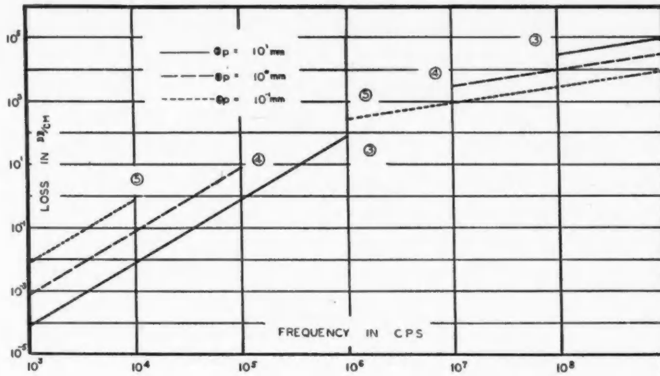

FIG. 3. Comparison between low and high frequency approximation for the absorption coefficient. Frequency range, 10^3–10^9 c/sec.

For $\nu > 10^7$ c/sec and $p < 1$ mm of mercury the conditions leading to Eq. (35) are met sufficiently well, and the same is true for $\nu > 10^2$ c/sec and $p = 10^{-5}$ mm of mercury. Under these conditions α varies effectively as $\nu^{\frac{1}{2}}$.

This treatment, of course, neglects the effect of heat conduction. The simplest way to introduce this effect in the general case is to go back to the treatment by Rayleigh,[6] where it is shown that owing to the combined effects of viscosity and heat conduction the particle velocity is given by

$$u \sim \exp(-x(z_1)^{\frac{1}{2}}), \tag{37}$$

where z_1 is the smaller of the roots of the quadratic

$$\begin{aligned} z^2(c^2/\gamma + 4i\omega\mu/3\rho_0) & \\ -z[c^2 + i\omega(\eta/c_v\rho_0 + 4\mu/3\rho_0)] & \\ \times i\omega c_v\rho_0/\eta - i\omega^3 c_v\rho_0/\eta = 0. & \end{aligned} \tag{38}$$

The various symbols in this equation have the same meaning as earlier in the paper. If we solve the quadratic under the approximation

$$\mu\omega/\rho_0 \gg c^2, \tag{39}$$

which is valid at high frequencies and low pressures, we obtain as a sufficient approximation to the required root

$$z_1 = (i\omega\rho_0/2\mu)(1 + 0.4i\rho_0 c^2/\mu\omega). \tag{40}$$

On this basis the absorption coefficient becomes

$$2\alpha = 8.04 \times 10^2 (\rho_0\nu)^{\frac{1}{2}}. \tag{41}$$

to an approximation that neglects $\rho_0 c^2/\mu\omega$ in comparison with unity.

[6] Reference 1, secs. 348 and 349.

The values of 2α from Eq. (41) are plotted in their appropriate regions of validity for various pressures in Fig. 3. For purposes of comparison, the classical low-frequency approximation values (from Figs. 1 and 2) are also plotted on the same graph. No attempt has been made to interpolate between the two regions. Of interest is the fact that at the higher frequencies the absorption coefficient varies as the square root of the frequency and also as the square root of the density. Hence the curves appear in reverse order as the pressure is decreased.

The results just obtained must not be taken too seriously as the following considerations make clear. Indeed, Eq. (41) implies that, for given ν, $\alpha \to 0$ as $\rho_0 \to 0$, a manifestly absurd result. We certainly must not employ Eq. (41) for too low densities. The difficulty encountered here results from the assumption that the viscosity is independent of the pressure no matter how small the latter is. This is probably true for pressures as low as 10^{-2} mm of mercury, but hardly for much lower pressures, as a reference to any standard text on kinetic theory of gases will confirm. As the pressure is decreased, the number of molecules per unit volume ultimately becomes too small to provide the transfer of momentum responsible for viscosity, and hence the viscosity itself will have to decrease. Presumably under these conditions μ will fall off at least as fast as the first power of ρ_0. The effect of this would be to keep τ relatively constant as ρ_0 decreases and hence to remove the ρ_0 from Eq. (41), 2α still varying approximately as $\nu^{\frac{1}{2}}$.

However, let us contemplate pressures for

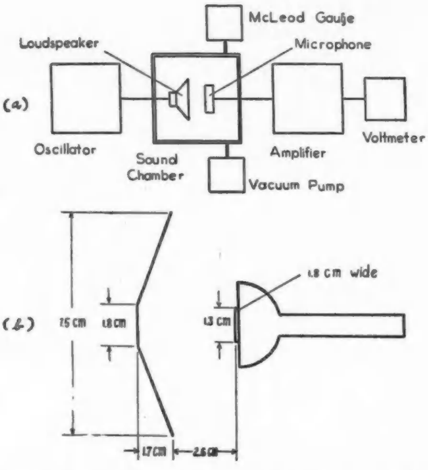

FIG. 4. (*a*) Schematic diagram of apparatus; (*b*) Orientation of microphone and loud-speaker.

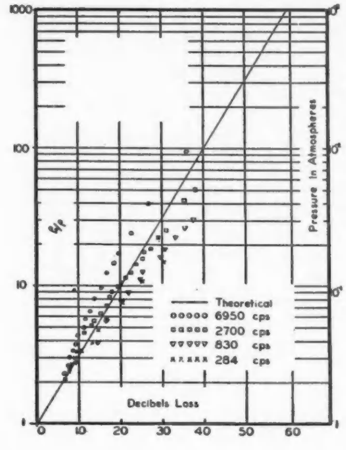

FIG. 5. Tank experiment. Transmission loss (in db) *versus* pressure.

which μ is still practically constant and frequencies high enough that $\omega\tau \gg 1$. Considering viscosity only for simplicity (these are order-of-magnitude considerations), we readily see from Eqs. (28) and (36) that

$$\omega\tau \approx 10^{-4} Vl/\lambda, \tag{42}$$

where V is the phase velocity and λ is the wave-length. From Eq. (16) we have, when $\omega\tau \gg 1$,

$$V = c(2\omega\tau)^{\frac{1}{2}}. \tag{43}$$

This finally leads to

$$\omega\tau \approx 25l^2/\lambda^2 \tag{44}$$

for air. Now when $l \approx \lambda$, we must conclude from elementary considerations, as was pointed out in 1917 by Schrödinger,[7] that the propagation of sound becomes impossible, since under these conditions it is on the average impossible to maintain differences in density and pressure in neighboring parts of the medium in the face of the increased rapidity of the exchange of molecules accompanying the lengthened mean free path. It is clear that there is only a very small pressure frequency range for which $l < \lambda$ and $\omega\tau \gg 1$. Hence the values of 2α given in the high-frequency region in Fig. 3 are probably unreliable, except in the sense that they confirm the very large attenuation in this range which the common-sense view dictates.

[7] E. Schrödinger, *Physikal. Zeit.* **18**, 445 (1917).

It would appear to be desirable to investigate the variation of viscosity with pressure at very low pressures in large spaces, that is, relatively unconfined mediums. The upper atmosphere immediately comes to mind in this connection. Most of the work on viscosity and heat conduction has so far been restricted to relatively small vessels in which surface effects ultimately predominate.

Measurement of Acoustic Transmission Loss in a Low-Pressure Tank

Few experimental data have as yet appeared in the generally available literature on the transmission of sound through air at low pressure, though several projects are underway and preliminary results have been reported recently.[8] We shall describe in this section the first results of an experimental program being initiated at Brown University.

In a metal tank of cylindrical shape (75 cm in diameter and 50 cm high), used originally as a metallic evaporation chamber, a loud-speaker and a microphone were set up very close together

[8] H. C. Rothenberg and W. H. Pielemeier, Pennsylvania State College, "Preliminary report on absorption in air from 10 to 100 kc/sec, manned by pulse techniques," and L. P. Delsasso, R. W. Leonard and R. Halley, University of California at Los Angeles, "The absorption of sound in air at reduced pressures;" papers delivered at the Washington meeting of the Acoustical Society of America, April 1948.

(Figs. 4a and 4b). Care was taken to insulate the mounting as far as possible from the walls of the tank. The loud-speaker was driven by a variable-frequency oscillator. In these preliminary experiments low frequencies—that is, under 7 kc/sec—were used exclusively. The chamber could be evacuated by a mechanical pump to a pressure of about 5 mm of mercury.

If we let I_a be the measured intensity of the sound received at the microphone (in arbitrary units) for given loudspeaker velocity amplitude at atmospheric pressure and standard room temperature, and I_p be the corresponding intensity for the same loudspeaker velocity amplitude at pressure p, we shall define the quantity

$$10 \log_{10}(I_a/I_p) \tag{45}$$

as the transmission loss in decibels at pressure p relative to atmospheric pressure. The experimentally determined values of this quantity at various frequencies from 384 to 6950 c/sec are plotted in Fig. 5. The solid straight line represents the quantity

$$20 \log_{10}(\rho_{0a}/\rho_{0p}), \tag{46}$$

where ρ_{0a} is the density of air at atmospheric pressure and ρ_{0p} the density at pressure p. The basis for expression (46) is simply that the radiation resistance for any transducer radiating into a fluid medium of density ρ_0 is directly proportional[9] to ρ_0. Hence other things remaining the same, the intensity of radiation varies directly with the mean density and hence the mean pressure of the medium. By the reciprocity theorem the receiving microphone obeys the same rule, which accounts for the factor 20. If we like we may consider the quantity in the expression (46) as representing the theoretically expected loss in the pressure chamber due to the impedance mismatch between transducer and air. As the pressure of the air is reduced it becomes progressively harder to get the energy from the transducer into the air and likewise from the air to the microphone.

From the general order-of-magnitude agreement between the experimental results and the curve based on expression (46) it seems clear that the pressure-tank measurements here reported indicate primarily the increasing impedance mismatch with decreasing pressure. The indicated shift in slope with frequency is suggestive but hardly significant at the present stage of the experiments. At the pressures and frequencies here considered actual absorption due to viscosity and heat conduction can hardly play much of a role, and this indeed is confirmed by the theoretical values already noted, where for $\nu=7000$ c/sec and $p=10$ mm of mercury the loss comes out to be about 4.5×10^{-3} db/cm.

To use the tank technic for the study of genuine absorption of sound at low pressure it will be necessary to go either to much higher frequencies or to much lower pressures or to both. This involves the construction of a new sound source and receiver outfit and the attachment of a faster, high-power pump.

The employment of the method also involves some theoretical points which must be examined with care for their effect on the results. These are the change in the source and receiver impedance due to the change in the air load at lower pressure and the effect of standing waves in the chamber. No account has so far been taken of the normal modes of the chamber.

Many interesting problems remain to be attacked, among them the influence of water vapor and other gases in air at low pressure. This will introduce a consideration of the extension to low pressures of the Knudsen-Kneser thermal-relaxation-time theory of sound absorption. Preliminary examination indicates that the lowering of the pressure will increase the expected absorption from this source. The transmission of sound waves of finite amplitude at reduced pressure will provide another challenging problem. The next few years will doubtless witness a vast increase in our knowledge of the propagation of high-frequency sound through highly rarefied mediums. This must go hand in hand with theoretical studies based on the kinetic theory of gases at extremely low pressure.

It is a pleasure to acknowledge the assistance of my colleagues Professor J. R. Frederick and Messrs. Carl E. Adams and E. F. Smiley in connection with the experimental measurements reported in this section, as well as certain of the calculations and curve plotting in the rest of the paper.

[9] G. W. Stewart and R. B. Lindsay, *Acoustics* (Van Nostrand, 1930) p. 247.

The Significance of Temperatures Derived from Emission Spectra

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A NUMBER of problems dealing with the physics of the earth's upper atmosphere, and many problems in the field of astrophysics, involve a quantity which is loosely called "temperature." A good deal of confusion exists regarding these temperatures, and it is important to realize that the word may have different meanings in different cases. It is the purpose of this article to explain the significance of the various temperatures quoted in astrophysical problems.

The concept of temperature in an atmosphere is complicated by departures from thermal equilibrium. If thermal equilibrium existed, the problem would be much simplified and all methods of computing temperature would give the same result. Consider what is meant by thermal equilibrium. Suppose we have an assemblage of molecules, atoms, and ions at a given temperature T. Collisions will take place between particles, and if the energy transferred during collisions is sufficient, molecules and atoms may be excited to higher energy levels, and a short time later unload this energy in the form of radiation. Other molecules and atoms may absorb the radiation; hence, the energy of the assemblage is being transferred from particle to particle through collisions and absorption and emission of energy. Stated more fully, atoms (or molecules) in a certain energy level A may absorb radiant energy and be raised to a higher level B. After a short interval these atoms (or molecules) return to level A and emit energy. Atoms (or molecules) in the lower level may also reach B by collision with other particles. In this case, the colliding particles rebound with lessened energy. Furthermore, atoms (or molecules) in the higher level may return to A by collision with particles. These particles rebound with increased energy. If the assemblage is in thermal equilibrium, then the number of absorptions from A to B equals the number of emissions from B to A, and the number of collisional excitations from A to B equals the number of collisional de-excitations from B to A. When this condition exists no spectral lines are formed, since energy levels are populated and depopulated at the same rate by both processes. Hence, the detection of spectral lines in an atmosphere is an indication of departure from thermal equilibrium.

There are a number of useful equations which may be applied when thermal equilibrium prevails. One of these is the familiar Maxwell velocity distribution law which gives the fraction of particles having velocities in a range between v and $(v+\mathrm{d}v)$,

$$f(v)\mathrm{d}v = 4\pi\left(\frac{m}{2\pi kT}\right)^{3/2} v^2 \exp(-\tfrac{1}{2}mv^2/kT)\mathrm{d}v. \quad (1)$$

In certain cases we may have a volume of gas which initially is not in thermal equilibrium. Collisions between the gas particles will gradually bring about a Maxwellian velocity distribution at a rate that depends upon the pressure, temperature, and the nature of the particles. For example, the diffuse material in interstellar space at a space density of the order of 3×10^{-24} g cm^{-3}, will reach a Maxwellian distribution in about 4×10^5 years. The same reasoning may be applied to the aggregation of stars known as the galaxy. The space density of stars in the neighborhood of the sun is 4×10^{-57} cm^{-3}, and stars the size of our sun will suffer an actual collision once every 2×10^{17} years. Hence, for these stars a Maxwellian velocity distribution will be reached only after a long period of time of the order of 7×10^{13} years.

The Boltzmann distribution law, which gives the distribution of atoms or molecules in various energy levels, may be applied when thermal equilibrium conditions exist. The law may be written

$$N_n = N_0\frac{\omega_n}{\omega_0}\exp(-\chi_n/kT), \quad (2)$$

where N_n is the number of atoms or molecules in level n, N_0 the number of atoms or molecules in the ground level, ω_n the statistical weight of level n, ω_0 the statistical weight of the ground

level, χ_n the excitation potential of level n and T the absolute temperature.

In an atmosphere consisting of atoms, molecules, positive ions, and electrons, only the electrons may be expected to approach a condition of thermal equilibrium. The reason is that atoms, molecules, and positive ions may absorb incident radiation and have an energy level distribution which is non-Boltzmann in character. Free electrons may experience elastic collisions with other particles, inelastic collisions which result in the excitation of atoms or molecules, collisions of the second kind with atoms or molecules in excited metastable levels, free-free transitions in the neighborhood of nuclei with the resulting emission of energy, or capture by ions with the emission of energy. Bohm and Aller[1] have considered the motions of the free electrons in the atmosphere of a planetary nebula, and conclude that the process of elastic scattering is by far the most probable. If this is so, the free electrons will reach a velocity distribution which is close to a Maxwellian distribution.

A third expression that applies to thermal equilibrium conditions is the Saha ionization equation, giving the relative numbers of ionized and neutral atoms (or molecules) in an assemblage at temperature T. Ionizations are brought about by collisions and absorptions of energy at a rate depending upon the temperature and the type of atoms (or molecules); recombinations take place at a rate depending upon the pressure. High temperature with low pressure favors the ionized condition. The Saha equation is

$$\frac{N_1}{N_0}P=\frac{2B_1(T)}{B_0(T)}\frac{(2\pi m)^{3/2}(kT)^{5/2}}{h^3}\exp(-\chi/kT), \quad (3)$$

where N_1 is the number of ionized atoms, N_0 the number of neutral atoms, P the pressure of the free electrons, and χ the ionization potential. The function $B_0(T)$ gives the distribution of the neutral atoms among the various energy levels. It can be computed for any temperature. Following the same notation, $B_1(T)$ is the corresponding function for the ionized atoms.

These equations which have been discussed briefly are used to determine temperatures of stellar atmospheres, of the earth's atmosphere, and of the interstellar material. It is more satisfactory, of course, to determine temperatures by measuring the total radiation emitted by a star, or by studying the intensity distribution of this radiation. Temperatures calculated by the first method are called "effective temperatures," since it is this quantity that is effective in producing the observed radiation from the star. As an example, we will now indicate how to compute T, the effective temperature of the sun. Stefan's law gives the total radiation per unit time from a unit area of the solar surface, or, in the usual notation, $E=\sigma T^4$, where σ has the value 5.672×10^{-5} erg cm^{-2} deg^{-4} sec^{-1}. The amount of solar radiation falling normally on unit area of the earth's surface per unit time is called the solar constant and has the value 1.35×10^6 erg cm^{-2} sec^{-1}. The total radiation emitted by the sun in unit time is $4\pi R^2F$, where R is the distance from sun to earth and F is the solar constant. If r is the radius of the sun, its surface area is $4\pi r^2$. Then by Stefan's law,

$$(4\pi R^2F/4\pi r^2)=\sigma T^4.$$

Substituting the known values of R, F, r, and σ, we find that T is around 5750°K. This method can be used only for those stars whose distances and sizes are known, data which are available in relatively few instances. In a few cases effective temperatures may be obtained from a study of the spectra of double stars.

A study of the intensity distribution in the spectrum of a star yields a quantity known as the "color temperature." This temperature is obtained by comparing the observed spectral energy curve with theoretical curves computed from Planck's law. The theoretical curve representing the equation which includes the stellar temperature will match the observed curve. This method of obtaining temperatures has limited applicability, because only in relatively few instances is it possible to determine the true spectral energy curve of a star. The continuous spectrum of all distant stars is distorted by the reddening effect of interstellar dust, and accurate corrections are difficult to make. A recent attempt[2] to determine the color temperatures of a

[1] D. Bohm and L. H. Aller, *Astrophys. J.* **105**, 131 (1947).

[2] W. Petrie, *Public. Dom. Astrophys. Observ.* **7**, No. 25 (1948).

number of Wolf-Rayet stars has encountered this difficulty.

In the absence of information on effective or color temperatures, it is sometimes possible to determine the temperature by applying Eqs. (2) or (3) to the intensities of certain emission lines in the spectrum of a star. For example, Eq. (2) may be used to compute a quantity called the "excitation temperature." The emission in ergs per cm^3 per sec in a spectral line is given by the expression,

$$E_{nn'}=N_nA_{nn'}h\nu_{nn'},$$

where N_n is the number of atoms in the upper level involved in the production of the spectral line, $A_{nn'}$ is the Einstein transition coefficient between levels n and n', and $\nu_{nn'}$ is the frequency of the radiation associated with a transition between levels n and n'. If thermal equilibrium conditions exist we may substitute the Boltzmann equation in the above expression. We then have

$$E_{nn'}=N_0\frac{\omega_n}{\omega_0}\exp(-\chi_n/kT)A_{nn'}h\nu_{nn'}. \tag{4}$$

Taking the ratio of the intensities of two spectral lines from the same atoms in the same condition of ionization, we may write

$$\frac{E_1}{E_2}=\frac{\omega_1}{\omega_2}\frac{A_1}{A_2}\frac{\nu_1}{\nu_2}\exp((\chi_2-\chi_1)/kT). \tag{5}$$

Substituting measured values of E_1 and E_2 and the quantities ω, A, ν, and χ from atomic data, we may determine the temperature T. This quantity is called the excitation temperature, since it refers to the absorbed radiation or the colliding particles which excite the atoms to the various energy levels. Aller[3] has used this method to compute temperatures in the atmospheres of a number of Wolf-Rayet stars, and Petrie[4] has determined the excitation temperature of the earth's upper atmosphere from the intensities of permitted oxygen lines in the auroral spectrum.

If the intensities of a number of lines are available, it is better to proceed in the following manner. Write Eq. (4) in logarithmic form and let $(N_0/\omega_0)h=C_1$. Then, $\log E_{nn'}=\log C_1+\log\omega_n-(5040\chi_n/T)+\log A_{nn'}+\log\nu_{nn'}$. In this form of the equation χ_n is in electron volts. Now let $\omega_nA_{nn'}\nu_{nn'}=D$. Then $\log(E_{nn'}/D)=\log C_1-(5040\chi_n/T)$. A plot of $\log E_{nn'}/D$ against χ_n produces a straight line, the slope of which gives the temperature. Goldberg[5] and Petrie[6] have used this method to determine the temperature of the solar chromosphere. However, if the Einstein A values for the spectral lines studied are not available, one may use an alternative expression for the intensity of a spectral line. This is,

$$E_{nn'}=\frac{N_0}{\omega_0}\exp(-\chi_n/kT)\frac{64\pi^4\nu^4}{3C^3}\frac{\epsilon^2a^2}{(4l^2-1)}s\rho^2. \tag{6}$$

In this equation ϵ is the charge on the electron, a is the radius of the first Bohr orbit, l is the larger of the initial or final values of the quantum number in the two electron configurations from which the line arises, s is the "strength" of the spectral line, and ρ is the radial quantum integral defined by

$$\rho=\int_0^\infty R_{nl}R_{n'l'}r^3\mathrm{d}r,$$

where R_{nl} and $R_{n'l'}$ are the radial wave functions. The factor ρ is a constant for all transitions between two electron configurations. If the spectral lines whose intensities are being compared belong to the same transition array, the factor ρ need not be included, since it is a constant for all the lines in this array. A large amount of data on theoretical s values is available, and Petrie (see reference 4, p. 42) has shown how to obtain these factors from measured laboratory line intensities. If the lines whose intensities are being compared belong to different transition arrays, then the ρ-factors must be included in the calculations. Menzel and Aller[7] have shown how these ρ-factors may be obtained in a number of cases. It is important to remember that the preceding equations apply only to an assemblage of particles in thermal equilibrium. Hence, if we use these equations with the intensities of lines arising from different

[3] L. H. Aller, *Astrophys. J.* **97**, 135 (1943).
[4] W. Petrie, *Can. J. Research* **A25**, 293 (1947).
[5] L. Goldberg, *Astrophys. J.* **89**, 673 (1939).
[6] W. Petrie, *J. Roy. Ast. Soc. Can.* **38**, 137 (1944).
[7] D. H. Menzel and L. H. Aller, *Astrophys. J.* **94**, 436 (1941).

energy levels, we obtain information on the degree of departure from a condition of thermal equilibrium.

The appearance in a spectrum of emission lines from the same element in two or more stages of ionization enables one to determine what is called the "ionization temperature." The number of neutral atoms of an element in a given energy level r may be obtained from one form of the Boltzmann expression,

$$\frac{N_{0r}}{N_0}=\frac{\omega_{0r}}{B_0(T)}\exp(-\chi_{0r}/kT). \tag{7}$$

In this equation N_{0r} is the number of neutral atoms in level r, N_0 is the total number of neutral atoms. Divide (3) by (7) and we have

$$\frac{N_1}{N_{0r}}P=\frac{2B_1(T)}{\omega_{0r}}\frac{(2\pi m)^{3/2}(kT)^{5/2}}{h^3}\times\exp(-(\chi-\chi_{0r})/kT). \tag{8}$$

The Boltzmann equation may also be applied to the ionized atoms. The number of ionized atoms N_{1s} in level s is

$$\frac{N_{1s}}{N_1}=\frac{\omega_{1s}}{B_1(T)}\exp(-\chi_{1s}/kT). \tag{9}$$

Forming the product of Eqs. (8) and (9), we have

$$\frac{N_{1s}}{N_{0r}}P=\frac{2\omega_{1s}}{\omega_{0r}}\frac{(2\pi m)^{3/2}(kT)^{5/2}}{h^3}\times\exp((-\chi+\chi_{0r}-\chi_{1s})/kT).$$

The relative intensities of the lines arising from these levels is

$$\frac{N_{1s}A_{1s}h\nu_{1s}}{N_{0r}A_{0r}h\nu_{0r}}=\frac{A_{1s}h\nu_{1s}}{A_{0r}h\nu_{0r}}\frac{2\omega_{1s}}{P\omega_{0r}}\frac{(2\pi m)^{3/2}(kT)^{5/2}}{h^3}\times\exp((-\chi+\chi_{0r}-\chi_{1s})/kT).$$

The electron pressure in dynes cm^{-2} is nkt, n being the number of free electrons per cm^3. This method has been used to compute temperatures in stellar atmospheres. The ionization temperature then, refers to the absorbed radiation or colliding particles which produce the observed equilibrium between ionized and neutral atoms.

Another quantity, the "electron temperature," may be obtained from the continuous emission that in certain cases appears at the limit of a spectral series. Consider the origin of this continuous emission. At high temperatures ionization and recapture processes are active. That is, atoms may absorb sufficient energy to lose one or more electrons. However, the resulting ions recombine with free electrons and become neutral atoms once again. For a given temperature, the Saha equation expresses the relative numbers of neutral and ionized atoms. Since the energies of free electrons are unquantized, the radiation resulting from the recapture process will form a continuous spectrum. If we consider the continuous emission at the limit of the hydrogen Balmer series, this emission being produced by the capture of an electron on the level $n=2$, the frequency of the emitted quantum is given by

$$h\nu=\tfrac{1}{2}mv^2+\tfrac{1}{4}hRc,$$

where $\frac{1}{2}mv^2$ is the kinetic energy of the free electron before capture, and $(\frac{1}{4})hRc$ the energy of level 2 measured from the ionization level, R being the Rydberg constant and c the velocity of light. The intensity distribution in this continuous spectrum depends upon the velocity distribution of the free electrons; hence measurements of the spectral gradient in this continuum determine the electron temperature, that is, the temperature which produces the observed velocity distribution among the free electrons. Menzel and Cillié[8] have used this method to calculate the temperature of the solar chromosphere.

In general, effective, color, excitation, ionization, and electron temperatures will differ for an assemblage of particles. Each temperature describes a different phenomenon, and each contributes to our information on the physical conditions of the assemblage. Electron temperatures in certain cases may be less than ionization temperatures. The latter are determined from the relative intensities of lines from the ionized and neutral atom. After the ionization takes place the free electron may experience inelastic collisions with other particles and give

[8] D. H. Menzel and G. G. Cillié, *Astrophys. J.* **85**, 88 (1937).

up part of its energy. Hence, the energy emitted during the recapture process will be characteristic of a lowered temperature. If the upper levels of an ion in the nth stage of ionization are populated by the recapture process, then the excitation temperature calculated from the spectrum of this ion will be much the same as the electron temperature of the free electrons captured by the $(n+1)$st ionization stage of the ion. In other words the excitation temperature from the spectrum of O II will be close to the electron temperature of the free electrons captured by the O III ion.

We have pointed out that for an assemblage of atoms, molecules, and ions in thermal equilibrium all methods of computing temperature give the same result. However, thermal equilibrium conditions are not found in the earth's atmosphere and in stellar atmospheres, and the choice of the various spectroscopic temperatures depends upon the physical phenomenon which one wishes to describe.

An Experiment with a Nonlinear Circuit

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CONSIDERABLE time is commonly devoted to laboratory work with linear electrical circuits in courses of instruction dealing with circuit theory. Nonlinear circuits are often dismissed with a few comments concerning the difficulty of analyzing them, even though many circuits of interest actually become nonlinear when large signals are applied. A relatively simple experiment involving forced oscillations in a nonlinear resonant circuit is described here. The experiment can be performed in a reasonable time with equipment usually available in a circuits laboratory. The nonlinear element of the circuit is an inductor with an iron core in which the magnetic flux is not a linear function of the current. The phenomena involved and the method of analysis are applicable to other resonant systems containing a nonlinear element, such as mechanical vibrating systems[1] or acoustical devices.[2] The method of analysis is standard and is described elsewhere.[3]

The circuit under study is shown in Fig. 1, and consists of an oscillator supplying a sinusoidal voltage of variable frequency to the series combination of the iron-core inductor and a fixed capacitor. The inductor must be chosen carefully, since the success of the experiment depends upon it. The core of the inductor should be of such design that it can be saturated magnetically with a relatively small current in the coil, so as to avoid the necessity of large power input to the circuit. At the same time, the self-inductance of the coil should not exceed a few henries, and the ohmic resistance of the winding should be small. Data given here were obtained using the primary winding of a Western Electric transformer, type D-163413, which is designed to operate at 400 cycle sec^{-1} with an input voltage of 115 volts. The iron core is relatively small in cross section. Ordinary 60 cycle sec^{-1} power transformers are not suitable unless a considerable amount of the core material is removed. Several small audio inductors have been found to function successfully in the circuit. The value of the capacitor should be such that the resonant frequency of the circuit is not too high, and the magnetic flux is therefore large. Voltmeters of high impedance are connected across the oscillator and across the inductor. An oscilloscope connected across the inductor is also useful, since it allows observation of the wave form of the voltage at this point.

If the driving voltage is maintained constant in amplitude, and its frequency is raised steadily from a low value, the rms voltage across the

[1] S. Timoshenko, *Vibration problems in engineering* (Van Nostrand, 1937), p. 137.
[2] H. F. Olson, *Elements of acoustical engineering* (Van Nostrand, 1947), p. 168.
[3] L. A. Pipes, *Applied mathematics for engineers and physicists* (McGraw-Hill, 1946), p. 600.

inductor varies as shown by the solid curve of Fig. 2. Circuit parameters used in obtaining this figure are $E_0=15$ volts rms, $C=1\mu\text{f}$, and the inductor described previously. The voltage E_L rises with the frequency until, at a certain critical frequency (about 230 cycle sec^{-1} for the figure), the voltage falls abruptly to a much smaller value, approaching the driving voltage asymptotically. If the frequency is now lowered steadily, again maintaining constant amplitude, the voltage across the inductor increases only slightly until another critical frequency (about 135 cycle sec^{-1}) is reached. At this frequency, the voltage jumps suddenly to the larger value of the curve obtained with increasing frequency. Between the two critical frequencies (indicated by the black squares of the figure) there is a range in which the voltage across the inductor may have either of two values, depending upon the past history of the circuit. Theoretically, a third value also exists in this range, but it is unstable and cannot be observed physically. The wave form of the voltage across the inductor becomes square-topped along the upper branch of the curve, for large values of this voltage. This distortion from sinusoidal wave form indicates the presence of a third harmonic component.

A qualitative explanation of the behavior just described may be based on the observation that the effective inductance of the coil becomes smaller when the amplitude of the current in it is increased. As the frequency of the driving voltage is raised from a low value, resonance is approached. The resulting drop in the impedance of the circuit allows an increase in current, giving a corresponding decrease in inductance and a rise in the resonant frequency. This process is cumulative, with resonance continually being approached but never quite reached. Eventually losses in the system become so large that the high current cannot be maintained, and the drop in amplitude occurs. The new current is based on the original large inductance. If the frequency

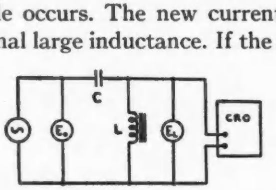

FIG. 1. Resonant circuit used in experiment.

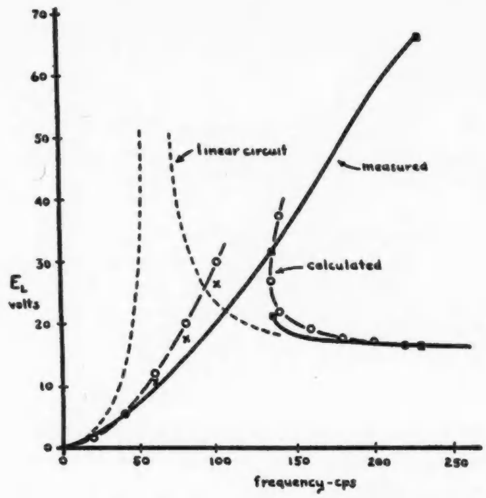

FIG. 2. Rms voltage across inductor of experimental circuit for constant driving voltage, $E_0=15$ volts.

is now reduced, resonance is approached once more, giving the rise in current and drop in inductance. Soon an unstable point is reached, such that a further rise in current would cause the inductance to become small enough to give a resonant frequency higher than the driving frequency, and the current jumps to the large value attained previously. The behavior of the circuit may be predicted by the approximate analysis which follows.

It will be assumed that the resistance of the circuit is small and can be neglected. This assumption simplifies the argument considerably, since the actual resistance of the circuit would be a nonlinear quantity itself. Neglect of the resistance prevents the location of the frequency of transition from the upper branch to the lower branch, however. Approximately, the relation between the current i in the inductor, and the magnetic flux linkage ϕ is

$$i=\phi/L_0+a\phi^3. \tag{1}$$

Hysteresis effects are neglected in writing this equation. The self-inductance would be L_0 if the coil were linear, and a is a coefficient determined by the amount of nonlinearity.

The voltage equation for the circuit of Fig. 1 may be written as

$$d\phi/dt+1/C\int i dt=E_1 \sin\omega t. \tag{2}$$

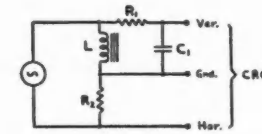

FIG. 3. Circuit for determining magnetization curve for inductor.

The successive terms represent the voltages across the inductor and the capacitor, and the driving voltage. If Eq. (1) is substituted into the derivative with respect to time of Eq. (2), the result is

$$d^2\phi/dt^2+\phi/L_0C+a\phi^3/C=E_1\omega\cos\omega t. \qquad (3)$$

The solution of this equation cannot be found readily. However, experimental observations indicate that the voltage across the coil (and thus, the flux in it) varies with time in approximately sinusoidal fashion. Furthermore, if resistance is neglected, the driving voltage and that across the coil must either be in phase or in phase opposition. Therefore, a first approximation to the steady-state solution might be

$$\phi=\phi_1\cos\omega t. \qquad (4)$$

Substitution of this solution into Eq. (3), together with the relations, $\omega_0^2=1/L_0C$, $b=a/C$, and $\cos^3\omega t=\frac{3}{4}\cos\omega t+\frac{1}{4}\cos 3\omega t$, gives

$$(-\omega^2\phi_1+\omega_0^2\phi_1+\tfrac{3}{4}b\phi_1^3)\cos\omega t+\tfrac{1}{4}b\phi_1^3\cos 3\omega t = E_1\omega\cos\omega t. \qquad (5)$$

If the inductor is nonlinear so that b is not zero, no value of ϕ_1 will satisfy this equation. The difficulty arises from the fact that Eq. (4) is not a correct solution for Eq. (3). However, it is reasonable to require that the fundamental oscillation satisfy Eq. (5) and to disregard the third harmonic term for the present. Such a procedure yields the relation

$$\tfrac{3}{4}b\phi_1^3=(\omega^2-\omega_0^2)\phi_1+E_1\omega. \qquad (6)$$

This cubic equation relates the amplitude, ϕ_1, of the flux wave to the circuit parameters and the driving voltage. Since the instantaneous voltage across the inductor is $e_L=d\phi/dt$, the rms voltage at this point is

$$E_L=2^{-\frac{1}{2}}\omega|\phi_1|. \qquad (7)$$

In order to make use of Eq. (6) for predicting the performance of the circuit, the constants for Eq. (1) are needed. These constants may be found from a magnetization curve for the inductor. Such a curve may be determined experi-

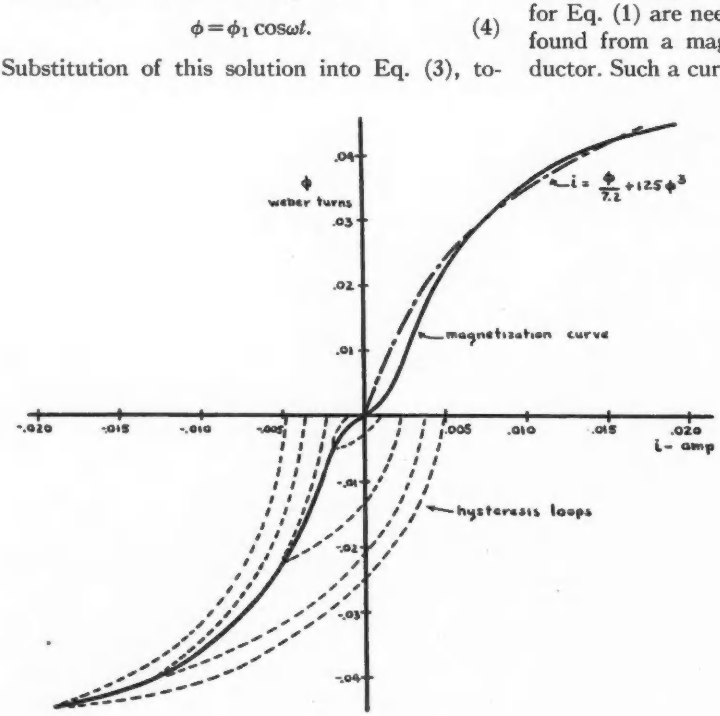

FIG. 4. Magnetization curve for inductor of experimental circuit.

mentally by the circuit of Fig. 3. The inductor is connected in series with a resistor, R_2, of relatively small value, and an oscillator of adjustable output voltage. The frequency is unimportant but should be fairly low. An integrating circuit, R_1 and C_1, is connected across the inductor. Typical values are $R_1=0.25$ meg, $C_1=0.5\mu$f, $R_2=1000$ ohms, and $f=45$ cycle sec^{-1}. The horizontal and vertical amplifiers of an oscilloscope are connected as shown in the figure. The horizontal deflection on the oscilloscope is proportional to the current, i, and the vertical deflection is proportional to the flux linkage, ϕ. It is necessary that the oscilloscope be calibrated so that the peak voltage required at the input terminals to produce a given peak deflection is known. Then the following relations apply:

$$i=e_H/R_2, \tag{8}$$

$$\phi=\int e_L dt=R_1C_1e_V, \tag{9}$$

where e_H and e_V are instantaneous voltages at the input terminals of the horizontal and vertical amplifiers, respectively.

The figure on the screen of the oscilloscope will be a hysteresis loop for the inductor. The loop will change in size and shape as the driving voltage from the oscillator, and the current in the coil, is changed. The tips of the successive loops will trace the desired magnetization curve. The curves for the D-163413 inductor are shown in Fig. 4. The complete magnetization curve is given by the solid line, while portions of several hysteresis loops are shown in the lower half of the figure. An approximation to the magnetization curve, as obtained from Eq. (1), is plotted in the first quadrant. The equation is too simple to allow the initial sharp curvature near the origin to be fitted, but a fairly good fit with the upper portion of the curve is possible. It is best to choose constant L_0 so the approximate curve has a slope at the origin about the same as the maximum slope of the actual curve. Constant a is then chosen so as to fit the knee of the actual curve fairly well. The equation used in Fig. 4 is

$$i=\phi/7.2+125\phi^3, \tag{10}$$

where i is measured in amperes and ϕ in weber turns.

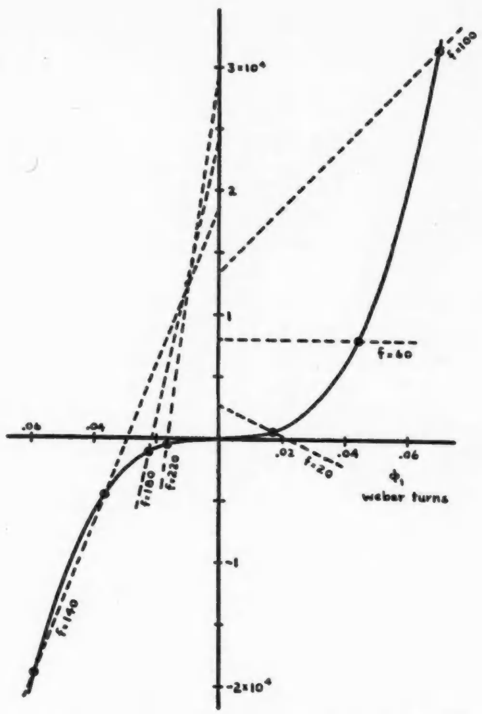

FIG. 5. Graphical solution for amplitude of flux linkage for experimental circuit.

A substitution into Eq. (6) of the known quantities for the experimental circuit gives

$$9.4\times10^7\phi_1^3=(\omega^2-1.39\times10^5)\phi_1+21.2\omega. \tag{11}$$

The flux and voltage in this equation are peak values. A graphical solution of the equation is shown in Fig. 5. The solid curve of the figure is the cubical parabola representing the left side of the equation. The dotted straight lines represent the right side of the equation for various frequencies, as indicated. The intersection of these lines with the vertical axis is determined by the second term of the right side, while the slope of the line is determined by the first term. Values of ϕ_1, found at the intersections of the solid curve and the dotted lines, are solutions of Eq. (11). For relatively low frequencies, only a single intersection occurs. Above a certain frequency, three intersections occur. The dotted line for 140 cycle sec^{-1} shows two intersections, and a third would occur if the figure were ex-

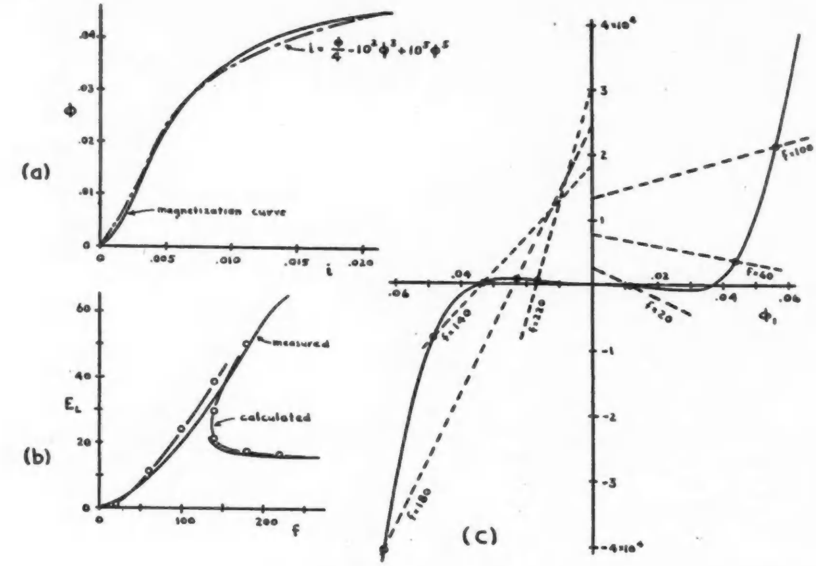

FIG. 6. Curves analogous to those of Figs. 2, 4, and 5, but using a more accurate approximation to the measured magnetization curve.

tended sufficiently. The rms voltage across the inductor at any frequency is given by Eq. (7), where the appropriate value of ϕ_1 is taken from Fig. 5. Points corresponding to values of E_L determined in this way are plotted as circles in Fig. 2. The calculated curve drawn through these circles is seen to agree fairly well with the measured curve. The portion of the right-hand branch of the curve having a positive slope is unstable. Therefore, the jump in amplitude at 135 cycle $\sec^{-1}$ as the frequency is lowered is predicted. Since resistance has been neglected, the drop in amplitude as the frequency is raised cannot be predicted.

The dotted curve of Fig. 2 is the calculated behavior if the inductance had the constant value, L_0. This curve may be determined from Fig. 5 by allowing the cubical parabola to degenerate to the horizontal axis. The curve for the linear circuit is very different from that for the nonlinear circuit.

The calculated and measured voltages for the nonlinear circuit may be brought into better agreement at the lower frequencies by a further correction. Equation (3) and Eq. (4) may be combined to give

$$d^2\phi/dt^2 = (E_1\omega - \omega_0^2\phi_1 - \tfrac{3}{4}b\phi_1^3)\cos\omega t - \tfrac{1}{4}b\phi_1^3\cos 3\omega t. \quad (12)$$

A substitution of Eq. (6) gives

$$d^2\phi/dt^2 = -\omega^2\phi_1\cos\omega t - \tfrac{1}{4}b\phi_1^3\cos 3\omega t. \quad (13)$$

Integration once with respect to time yields

$$e_L = d\phi/dt = -\omega\phi_1\sin\omega t - (b\phi_1^3/12\omega)\sin 3\omega t. \quad (14)$$

The first term is the first approximation of the fundamental frequency, already used. The second term is a correction term having the third harmonic frequency. Its rms value is

$$E_{L(3)} = 2^{-\frac{1}{2}}b\phi_1^3/12\omega. \quad (15)$$

The phase relations indicated by Eq. (14) are such as to give a square-topped wave. Approximately, the peak of e_L is less than the peak of the fundamental component by the peak value of the third harmonic. The vacuum tube voltmeter used in measuring E_L, as plotted in Fig. 2, measured peak value although its calibration was in rms value. Therefore, its reading was too low if only the fundamental component is con-

sidered. Values of $E_{L(3)}$, as found from Eq. (15), may be subtracted from the values of E_L found from Fig. 5 and Eq. (7), to give more nearly the values indicated by the voltmeter. These corrected points are plotted as crosses in Fig. 2.

The region of largest difference between measured and calculated curves of Fig. 2 is along the upper branch where the point of discontinuity is approached. In this region the value of ϕ_1 is so large (see Fig. 5 for 100 cycle sec^{-1}) that the approximate curve of Fig. 4 is no longer close to the actual magnetization curve. A better agreement between these curves may be obtained by the addition of a third term to Eq. (1), so that it becomes

$$i=\phi/L_0+m\phi^3+n\phi^5. \tag{16}$$

The three coefficients of Eq. (16) allow it to fit the measured magnetization curve more closely than was possible with Eq. (1), but considerably more mathematical work is now required to find these coefficients. They must be chosen carefully or a poor fit will result. Successive terms of the equation are large and of opposite algebraic sign, while their sum remains small. A poor choice of the coefficients causes the curve of Eq. (16) to cross the magnetization curve at certain chosen points, but to oscillate violently either side of the curve between those points. A fairly good fit between the curves is shown in Fig. 6(a), where the equation is

$$i=\phi/4-10^2\phi^3+10^5\phi^5. \tag{17}$$

It is not difficult to show that, following the previous analysis, the amplitude of the flux linkage is now given by

$$3m\phi_1^3/4C+5n\phi_1^5/8C=(\omega^2-\omega_0^2)\phi_1+E_1\omega. \tag{18}$$

The solution for ϕ_1 may be carried out graphically as before. This process is shown in Fig. 6(c), and the resulting curve of voltage across the inductor is shown in Fig. 6(b). No additional correction has been applied, but the calculated and measured curves here are seen to agree more closely than do those of Fig. 2.

The behavior of the circuit of Fig. 1 is actually somewhat more complicated if small driving voltages are considered. With very small driving voltage, the inductor is nearly linear and a simple resonance peak is observed. However, the effective value of the inductance at very small currents is considerably less than the constant L_0 of Eq. (10) or Eq. (17). In fact, this inductance is given by the slope of the magnetization curve of Fig. 4 at the origin, and is less than one henry for the coil used here. The resonant frequency with small currents is therefore much higher than the ω_0 of the analysis. As the driving voltage is increased, operation swings up the rising portion of the magnetization curve, and the effective inductance becomes larger, thereby lowering the resonant frequency. Still further increase in the driving voltage causes operation over the knee of the magnetization curve, and the phenomena previously described take place. An analysis of the complete behavior would require still more terms in Eq. (1), and the entire procedure would be much more complicated.

In spite of the simplifying assumptions used in the experiment, it serves to illustrate the measurement and analysis of a typical nonlinear resonant system and leads to results which are fairly good. The techniques of measuring the magnetization curve for the iron-cored coil, and the graphical solution of the equation for the amplitude, are interesting tools for other purposes. The performance of the experiment and the necessary calculations can be carried out in a reasonable length of time.

The instinctive weapon of the bourgeois, the reading and writing class, is the book. Whenever he is in a state of danger or hope he starts furiously to learn.—William Bolitho.

First-Line Problems of Graduate Study

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IT seems to me that anyone with the temerity to suggest improvements in the handling of graduate study had best start by examining his own qualifications. This is especially true because the research professor and the dean are likely to see the over-all problems of graduate study from very different perspectives. Administration representing organization, and teaching, which symbolizes inspiration, are more widely separated in graduate than in undergraduate education. If these natural divergences have meshed without conflict in my own thinking it is, perhaps, because I have had the opportunity to complete the cycle of graduate teaching, graduate administration and graduate teaching again. It has been no small contribution to my education to return during the past two years to teaching and research where it has been possible in an impersonal way to observe the workings of one of the larger graduate schools of engineering. It is unfortunate that so few presidents, deans or professors have the opportunity to complete this cycle of observation. It is a sobering and calming experience.

If a re-inspection of graduate study is to prove useful, it should search out the weaknesses of graduate education and strengthen them. But weakness or strength depends upon the end being sought. A practical viewpoint can be a weakness if the objective of graduate study is mathematical facility, but it may be a mark of strength in a curriculum designed to forward a constructive art. And surely it must be agreed that the broad sweep of the physical sciences and their applications through engineering include the entire range from abstract theory to the most applied type of design. Of course, a part of these studies belongs to undergraduate education and another part may be found only in design offices. In any case, graduate study cannot confine itself wholly to the mathematical or theoretical studies even including experiment. It must also make provision for the less routine or higher aspects of design investigations.

When the objectives of graduate study in any particular department are being settled, the greatest influence thereon should be exerted by the interests of the teachers. There is no reason why a department of physics, for example, must teach a graduate course in spectroscopy. The decision should depend upon whether there is a teacher on the staff who has a special interest and ability to teach such a course; and my criterion would be whether research in spectroscopy was actually under way in that department. I find no difficulty in commending a graduate school in civil engineering devoted exclusively to hydraulics without the offering of a single course in structures, or vice versa. Students should select a graduate school on the basis of its reputation for highly specialized studies.

You will see then that I am not in sympathy with the idea that graduate study may be just a fifth year of education in a broad field. There is much in common in my thinking about a fifth year and a fifth wheel. We start a group of freshmen that includes within it (a) future scientists, or specialists, (b) future technological salesmen, promotors and managers, (c) future construction and production men, and (d) many young men who think they would rather study science or technology than liberal arts, commerce or agriculture. For three of these four groups, four years of study is the equivalent of four wheels on a cart—it serves all possible purposes. For some, three years or three wheels provide an adequate educational vehicle. For others a two wheeled education serves just as a two wheeled cart will transport necessities. Even the man with one year of college education has at least the aid of an educational wheelbarrow which is far superior to loading one's back!

No doubt some one will point out that the fifth wheel was an important invention when added to the wagon since it permitted the operator to *guide* the vehicle. And this too seems to me an apt analogy in education. It is worse than wasted to add a fifth wheel just anywhere on the vehicle. But when scientifically placed in a horizontal plane it adds a new quality of direction or

guidance, the perfect symbol of vision or leadership in science and technology. Leadership in science develops only from deep knowledge gradually broadened through experience. A general fifth year of broadening education universally required in all technological curricula (and commonly preceeding the junior year) might be likened to putting a five wheel cart in front of a horse without a driver. Nevertheless, if adopted only here and there, such a fifth year might meet the needs of limited numbers of students who would then select such curricula because of their special interests. I criticize only the universality of the idea as included in the common question "When will we come to five-year curricula in engineering and science?"

In June 1945 the *Journal of Engineering Education* printed the Manual of Graduate Study in Engineering, a 37-page report of a three-year continuous study by a committee of experienced graduate educators. As I reread this manual it seems to me that it is as applicable to any field of science as it is to engineering because any committee of competent educators must inevitably come to about the same conclusions. Without being didactic it tries to outline good procedures and desirable objectives concerning the matters of admission requirements, degree requirements, major and minor studies, the thesis, language requirements, mathematics, non-technical studies, undergraduate courses for graduate credit, required examinations, and so forth. In review I still find that it performs a very useful service.

Nevertheless, I also find that the manual over-emphasizes the techniques or the detailed organization of graduate study in relation to the less tangible but more significant factors. This is natural because it is easy to be definite and to write definitely about the need for written and oral examinations while it is difficult to think about and still more difficult to write about the intangible quality of inspiration of students by the faculty. Nevertheless, the latter is important, and the kind of examination used is trivial by comparison. If the examination becomes ineffective, as I have observed occurring in my own institution, it can be made effective by changing its form, by transferring its position in the curriculum, or by using a different system of selecting the examination committee. But what is to be done if students are not being inspired by the faculty in a whole department or school? How are new faculty members to be selected? Is mediocrity to be permitted to perpetuate mediocrity as it will, inevitably? Are young staff members to be encouraged, stimulated and rewarded for accomplishments in research? Are the few, the very few, great graduate teachers to be rewarded in proportion to their distinction or are our institutions to gamble on their loss in order to maintain a balance in institutional salaries and teaching loads? In the answers to such questions lie the more significant influences upon graduate study. Such answers lead to one institution becoming great in the national sense while another remains effective merely in its own locality.

If an institution is interested exclusively in undergraduate education, or nearly so, it can avoid solving many of these embarrassing problems. But the graduate institution must face these issues squarely. Administrative officers carry a large share of the responsibility. A great president will attract great scientists to his institution and then he will contrive to have them shine as brilliantly as himself. An effective dean will be known by the reputations of his key teachers. Beware of any institution which is known to have a brilliant president or dean but an unknown faculty. There are even department heads who avoid the competition of brilliant associates; but the best administrators seek such competition as the life blood of an institution.

After a distinguished graduate teacher and researcher or a promising younger scientist is attracted to the institution, what further effort should be made to assure his continued research productivity, without which his teaching will soon become stale and eventually will stop being fruitful? Here is the place where education has the least reason to be congratulated. I recall reading that the investment in tools for a factory worker averages about $6000 and reaches $50,000 in some industries. If such an investment is needed to make an average workman productive, is it not reasonable to expect a scientist to require an investment many times as great? The institution that hires a graduate professor and assumes that he can be made pro-

ductive by providing him with a private office and little else is due for disappointment.

The theoretical analyst usually needs one or more assistants who are more than inexperienced students. He may need calculating machines of a simple or complex nature. He always needs secretarial service that is not merely the right to borrow the help of another person's secretary. With less than these his productivity will be gradually strangled. Yet how many universities really provide this minimum standard? The experimentalist requires a laboratory and a shop with laboratory assistants and a mechanician. Everyone has seen professors, young and old, making their own equipment at the expense of time that cannot be purchased. To the industrial research director this self-service merely looks so foolish that he doubts its existence. Industrial research laboratories are overstaffed with hands and understaffed with brilliant minds. Relatively speaking, the converse is true in educational research.

How many educational institutions provide a service whereby a professor can get high quality drafting done without taking time to train the draftsman himself? If a professor does not have full time need for a mechanic, an electrician or an instrument maker, can he obtain such services from a central service bureau or must he beg the loan of a workman from an associate who runs a laboratory, or from the buildings and grounds department? Industrial people cannot understand why we show such lack of foresight in improving our research output.

The war years produced at least one good result in educational research. The availability of government funds was used for the first time in many institutions to provide collateral services to research workers. Postwar contracts have helped to keep such services in existence. Also, the exchange of workers between education, industry and government laboratories served to educate college administrators and led research workers to insist upon more aid in research. Nevertheless, this lack is still predominant, and it reflects little credit upon the efficiency of educational operations.

It is such broad questions that lie at the heart of the problem of producing effective leadership in graduate study and research. We are always short of brilliant minds. Let us try to make those that education is lucky enough to have as effective as possible by providing them with the necessary tools and assistance. Only in this way can the hours be freed that are so badly needed for real direction of graduate study and research.

Lloyd William Taylor

We regret to have to record the tragic death of Lloyd William Taylor, Professor of Physics at Oberlin College, who was killed in a mountaineering accident on Mt. St. Helens, Washington, on Sunday, August 8, 1948. To commemorate in appropriate fashion his notable contributions to physics and to the American Association of Physics Teachers, a Lloyd William Taylor Memorial number of the American Journal of Physics is being planned for the early part of 1949.—T. H. O.

A Null-Deflection Magnetometer with Electromagnetic Control

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THE couple exerted on a coil with its plane in a magnetic field is directly proportional to the current in it. If an external torque be applied to such a coil, causing it to turn, it follows that the current required in the coil to restore it to its original undeflected position is proportional to this external torque. Hence it should be possible to convert a center-zero moving-coil detector galvanometer, such as is usually employed in a Wheatstone bridge circuit, so that it could be used either for measuring small forces or torques or as a null-deflection magnetometer.

The purpose of this article is to explain how the latter aim has been accomplished and to describe some experiments carried out with the magnetometer. It is hoped that it may serve to stimulate the interest of those teachers and students who have the desire and the time to make and use apparatus of a somewhat unconventional kind, for in so doing much valuable knowledge and technical skill can be acquired. The particular instrument described here is somewhat fragile and rather large, but there is no reason why improvement could not be made in these respects by the choice of other and perhaps more suitable components and materials. The magnetometer has a particular advantage in that its operation is independent of the direction of external magnetic fields, provided they do not vary during the time of the experiment.

Details of the Magnetometer

The moving-coil instrument selected for use had a pointer with a center zero scale. It was 6.5 cm high and its 10-cm diameter dial was calibrated 5–0–5 ma. It was an old instrument and its maker could not be identified.

It was first tested to see whether the currents needed to restore the pointer to its zero mark were directly proportional to the loads applied to the end of a thin aluminum rod fixed by paraffin wax to the coil mechanism so that it projected beyond the case of the instrument, opposite to and in line with the pointer. The weight of the aluminum rod was counterpoised by fixing lead shot to the pointer by paraffin wax. The heated blade of a small screwdriver was found to be a convenient tool for manipulating the wax. The instrument was fixed to a board so that its pointer moved in a vertical plane.

For calibration purposes, six riders were cut from copper wire and weighed. Each was applied as a load at a fixed point on the aluminum rod and the current required to bring the pointer back to the no-load mark was recorded. The values of load and current given in Table I and shown graphically in Fig. 1, show clearly that these quantities are directly proportional.

In this form the instrument could well be used to measure forces ranging from 1 to 23 dynes. There is no reason why a more sensitive balance could not be made by choosing a more suitable coil movement.

The conversion of the chosen movement into the magnetometer used in these experiments was next carried out. The arm h replaced the thin aluminum rod already mentioned. It was made by drawing out a glass tube and bending it to the shape shown in Fig. 2. It was fastened to the movement C by means of paraffin wax as before.

Two small Alcomax 2 magnets[1] M_1, approximately $5\times1\times1$ mm, were mounted at the end of arm h, also by wax. A concave mirror m of radius 100 cm was fixed to the arm over the axis of rotation of the movement.

The load added by these parts was counterpoised by means of lead shot L stuck onto the pointer P by wax. The final adjustment for

TABLE I. Restoring current and mass of rider.

Rider Length (cm)	Rider Mass (mg)	Restoring current (ma)
1.0	4.0	0.84
2.0	7.5	1.61
3.0	11.5	2.43
4.0	15.1	3.20
5.0	19.0	4.03
6.0	22.7	4.81

[1] Supplied by Messrs. W. Jessop & Sons of Sheffield.

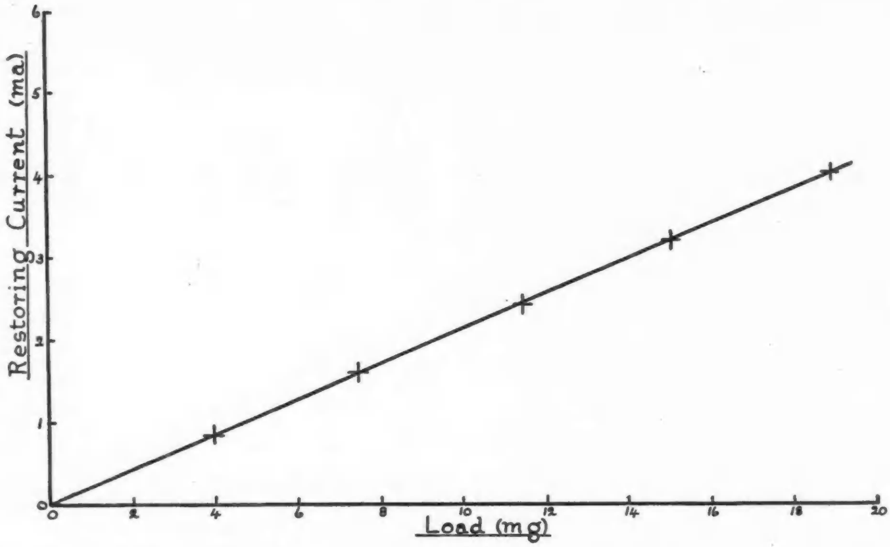

FIG. 1. Linear relationship between the current required to restore the moving coil to its undeflected position and the load applied to a fixed point on the arm attached to the coil.

balance was made by moving the aluminum rod A in or out of the end of the hollow glass arm h, to which it was finally fixed also by wax. The balance thus obtained was correct for both vertical and horizontal movements of the moving system.

The arm h was about 32 cm long and was made as light as possible yet rigid enough to prevent sag under the load of the magnets M_1 and also to prevent whip when subjected to the normal vibrations of a laboratory bench.

Figure 3 shows in plan and elevation how the magnetometer was mounted in a small cabinet with a hinged lid, fitted with a glass front and side. The cabinet was 23 cm wide, 27 cm high and 55 cm long. The box was essential to eliminate the effect of air currents. It should be noted that the movement was now in a horizontal plane.

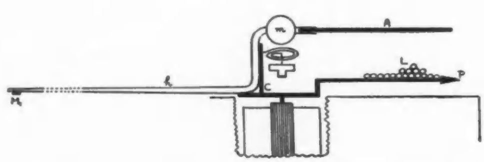

FIG. 2. Method of attaching the small magnets to the coil movement.

The box was fitted with plane mirrors X and Y to enable the deflecting magnets to be lined up correctly on the axis of the magnets M_1. A hole H was cut in the lid of the box so that before each experiment the no-deflection position of M_1 could be checked and reset if necessary by the zero control on the movement; however, no such adjustment was found to be necessary. The distance between the center of M_1 and the glass front was accurately measured.

The restoring current required to bring the deflected magnets M_1 back to the no-deflection mark was controlled and measured with the circuit given in Fig. 4, the terminals of which are connected to those of Fig. 3. In Fig. 4, G is a Cambridge Versatile galvanometer of resistance 500 ohm, reading to 120 mv, and shunted by a resistance box S; R is another resistance box and C is a commutator; E is a 2-v storage cell and R_2 a 19-ohm, 2.8-amp rheostat used as a potential divider. The resistances of S and R_1 were adjusted so that for a given experiment the current was entirely controlled by the movement of the slider of R_2.

The damping of the magnetometer was such as to cause no delay in taking readings and when the key K was closed it was considerable.

Experiments Performed

(1) Force between a very small magnet and a short magnet placed on a common axis

Theory of experiment.—If the magnetic field strength at P (Fig. 5) be H, then the force on the very small magnet of length dx_1, pole strength m_1 and moment M_1 is given by

$$(dH/dx)dx_1\cdot m_1=(dH/dx)M_1,$$

and thus

$$(dH/dx)M_1h=bi, \tag{1}$$

where b is the couple per unit current acting on the coil of the movement which is carrying current i to maintain no deflection.

For a magnet of moment M and magnetic length $2l$,

$$H=2Mx/(x^2-l^2)^2$$

and

$$dH/dx=-2M(3x^2+l^2)/(x^2-l^2)^3.$$

If the magnet is small, so that l^2 is negligible compared with x^2,

$$dH/dx=-6M/x^4,$$

whence the force on M_1 due to M is $6MM_1/x^4$. Thus

$$6MM_1h/x^4=bi,$$

whence

$$ix^4=\text{const.} \tag{2}$$

and

$$\ln i+4\ln x=\text{const.} \tag{3}$$

Experimental details.—The Alcomax 2 magnet M (Fig. 5) had dimensions $4\times1\times1$ cm and was supported on a platform made by mounting a meter stick on wooden supports at such a height that it lay with its axis along the axis of the small Alcomax 2 magnets M_1 on the magnetometer. The correct positions of the platform and the magnet M were found by means of the images reflected in the glass front and in the vertical mirror on the remote side of the magnetometer case.

Values of restoring current i were measured for values of x between 13 and 19 cm, and for both polarities of magnet M. The experimental values of ix^4 differ from the mean by 3 percent or less.

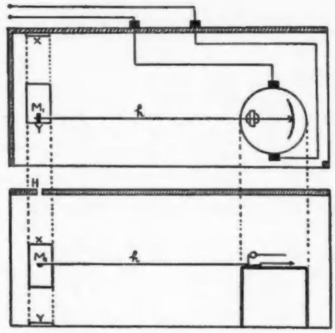

FIG. 3. Plan and elevation of arrangement used in mounting the magnetometer in its cabinet.

(2) Force between a very small magnet and a single pole placed on the axis of the magnet

Theory of Experiment.—The field strength at P (Fig. 6), due to the Robison magnet NS of pole strength m, is

$$H=\frac{m}{x^2}-\frac{m}{(x^2+L^2)}\cos\theta$$

$$=\frac{m}{x^2}-\frac{mx}{(x^2+L^2)^{3/2}},$$

and

$$\frac{dH}{dx}=-\frac{2m}{x^3}-m\left[\frac{L^2-2x^2}{(x^2+L^2)^{5/2}}\right].$$

For values of x which are small compared with L the second term may in the right-hand member be neglected in comparison with the first, whence the force on M_1 due to the single pole m is

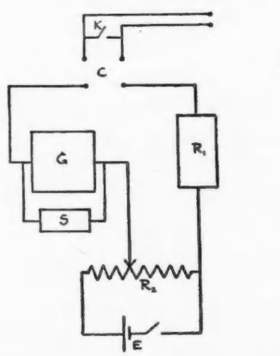

FIG. 4. Circuit used for controlling and measuring the restoring current.

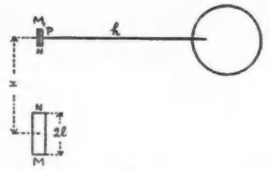

FIG. 5. Arrangement of the deflecting magnet and the magnetometer in Experiment (1).

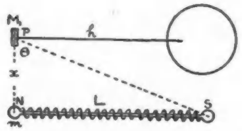

FIG. 6. Arrangement of the Robison magnet and the magnetometer in Experiment (2).

$2mM_1/x^3$; thus

$$\frac{2m}{x^3}M_1h=bi$$

and

$$ix^3=\text{const.} \tag{4}$$

or

$$\ln i+3\ln x=\text{const.} \tag{5}$$

Experimental details.—The Robison magnet NS (Fig. 6) was 44 cm long and had pole m supported on the wooden platform already mentioned; to obtain a constant and large value of the strength of this pole the magnet was placed in a solenoid 41 cm long wound on glass and having two layers of turns of No. 24 S.W.G., double cotton covered copper wire wound 12 turns to the centimeter.

A current of 3.5 amp was found to be sufficient to saturate the steel of the magnet; it was reduced to 0.5 amp during the experiment. It was also found that 0.5 amp produced no deflection on the magnetometer when the solenoid was used without the magnetic core, even with the smallest values of x. The values of x used lay between 10 and 15 cm.

Experimental values of ix^3 were again found to be very nearly constant.

(3) Magnetization curve and hysteresis cycle

Experimental details.—The specimen used was a knitting needle K (Fig. 7) to which was soldered a copper handle H. It was placed, after demagnetizing it in a separate alternating-current circuit, in the solenoid L used in Exp. (2), with one end opposite the magnetometer magnets M_1.

A 21-ohm, 4 amp rheostat was used as a potentiometer with an 18-v battery; the ammeter A had a range of 5 amp and the current was reversed by means of the commutator C.

The effect of the solenoid was considered negligible since when it was carrying 4 amp it caused a deflection of only 2 divisions and no deflection at all with 1 amp as compared with a deflection of 120 divisions due to the needle K when it was saturated.

Great care had to be taken in that part of the cycle when the polarity of K was opposite to that of M_1, since the position of no deflection of M_1 was then one of unstable equilibrium.

Readings of the solenoid current and the restoring current to bring the magnetometer back to its initial no-deflection position were taken over a complete cycle, the former being proportional to the magnetizing force and the latter to the intensity of magnetization. The results are shown graphically in Fig. 8.

(4) Variation in intensity of magnetization with temperature

Experimental details.—A small bar magnet of dimensions 5.0×0.75×0.6 cm was placed with its axis in line with that of the magnets M_1 of the magnetometer and was heated in the furnace F shown in Fig. 9. The furnace was made by replacing the five burners in a standard tube furnace, 15 cm long, 10 cm wide and 18 cm high.

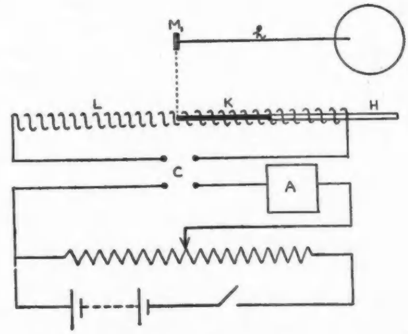

FIG. 7. Arrangement of the specimen, the magnetizing solenoid and the magnetometer together with the circuit used to control and measure the magnetizing current in Experiment (3).

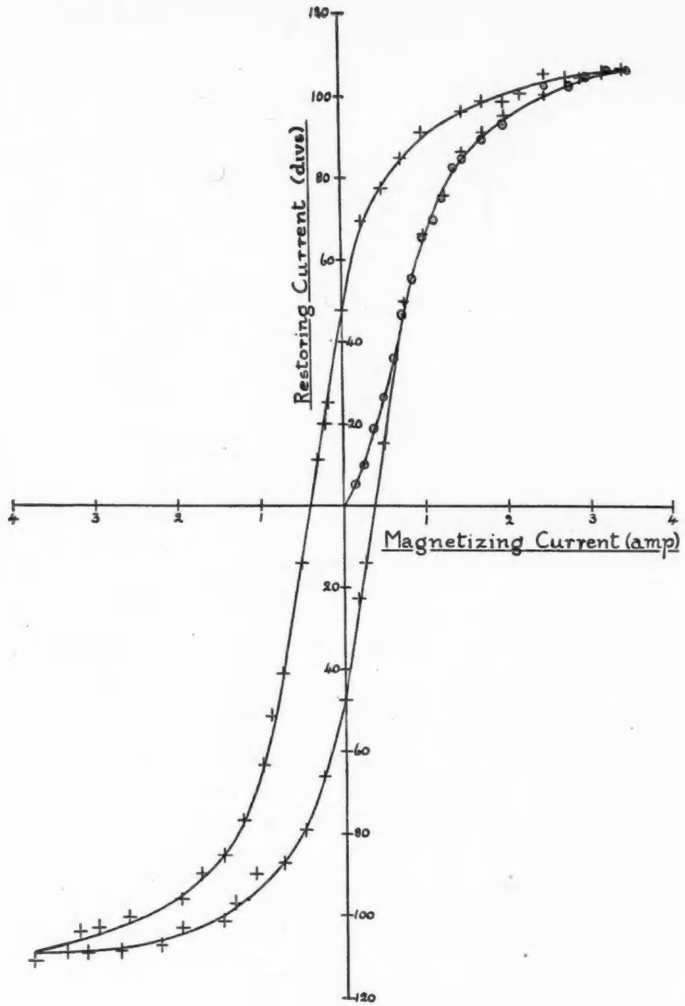

FIG. 8. Representation of the magnetization curve and hysteresis cycle for the steel knitting needle used in Experiment (3). The restoring current is proportional to the intensity of magnetization and the magnetizing current to the magnetizing force.

by a single copper tube C, 34 cm long and 1.5 cm in internal diameter, drilled at one end with four rows of six holes each to serve as jets after the manner of a gas poker; the tube was supported in a clamp at the opposite end and gas was admitted by a bunsen burner B as shown and the gas flow was controlled by a screw clamp K.

The end of C was plugged with asbestos wool A and the ends projecting through the firebrick of the furnace were passed through asbestos disks, thus keeping the heat at the end facing the magnetometer from its glass front. The distance x was about 17 cm.

The temperature was controlled by the size of the flame jets and by putting on or taking off the three firebrick covers D; it was never necessary to make the flame jets very large.

The intensity of magnetization of the steel of the magnet was proportional to the restoring current used with the magnetometer. The temperature was measured with an iron-constantan thermocouple XY connected in series with a resistance box R and a Cambridge Versatile

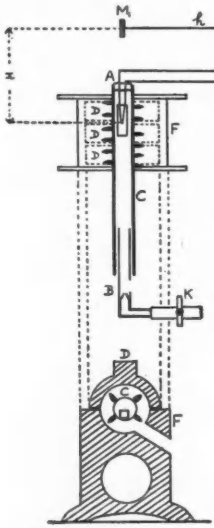

FIG. 9. Arrangement of the deflecting magnet, the gas furnace, the thermocouple and the magnetometer used in Experiment (4).

galvanometer G of 10-ohm resistance and 2.4-mv range.

Results are shown in Fig. 10 for two new magnets. Both magnets were made of the same steel but its exact nature was not known.

Comments.—It is interesting to note the reversal of polarity at W for the first magnet and to attempt an explanation of it. (For the second magnet the temperature was reduced before the critical value was reached.)

If it be assumed that for the steel of this magnet the permeability increases rapidly to a maximum as the critical temperature is approached and then falls quickly to unity at the critical temperature for small values of the magnetizing force, as is the case for iron,[2] then the magnetism induced in the bar magnet by the magnets M_1 of the magnetometer might be sufficient to overcome that due to the earth's field together with the permanent magnetism remaining after the heating to which the bar had already been subjected.

If the temperature had been further increased, the curve beyond the point Y (Fig. 10) would presumably have followed the shape shown by the dotted line, ending by meeting the temperature axis at Z, the ferromagnetic Curie point.

It should be pointed out that the horizontal component of the earth's field made an angle of about 65° with the axis of the bar magnet and was in such a direction as to cause it to retain its original polarity, the effective magnetizing force thus being $H_0 \cos 65°$.

On cooling from the point Y the reversed polarity acquired as suggested above would be retained but would decrease, assuming that the permeability decreased with fall in temperature

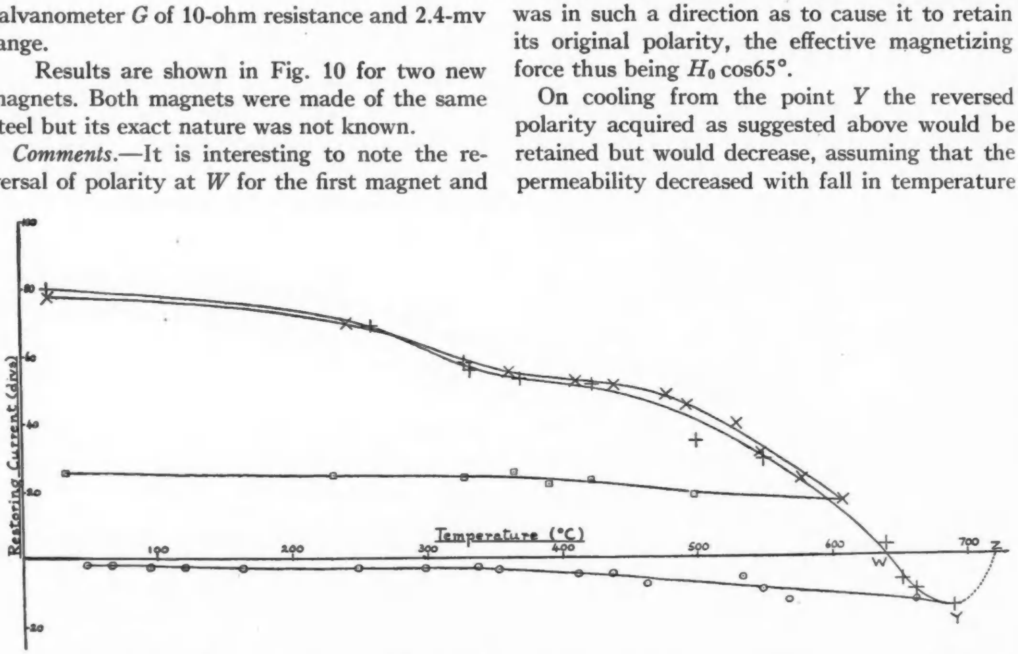

FIG. 10. Variation of the intensity of magnetization with temperature for the two new steel magnets used in Experiment (4). The restoring current is proportional to the intensity of magnetization. +: Magnet 1, temperature increasing. ⊙: Magnet 2, temperature decreasing. ×: Magnet 3, temperature increasing. ⊡: Magnet 2, temperature decreasing.

[2] R. M. Bozorth, *Rev. Mod. Physics* 19, 57 (1947).

as it does for iron in weak fields. Again if it be assumed that the field of the magnets M_1 in the region of the bar magnet was comparable with the effective field of the earth, $H_0 \cos 65°$, then as these were in opposite directions the previous explanation of the reversal of polarity at W as due to induction is unsatisfactory.

If, however, the bar magnet had been originally magnetized with polarity opposite to that possessed by it at the beginning of the experiment, then on heating it might reverse its polarity when nearing the critical temperature owing to the persistence of some of the more stable domains, oriented in their original direction, after the disappearance of the less stable domains oriented by the subsequent and reversed magnetizing force.

This explanation was suggested on reading of the problem of providing stability in the magnetization of vessels after being subjected to the process of "wiping" or stroking with a magnet or conductor carrying a current, developed in wartime for the purpose of changing the original magnetization in the steel hulls of ships as a means of protection against magnetic mines.[3]

Although the former explanation is the more likely, as the magnet was a new one, it may be that the reversal of polarity in question was in some measure due to both causes.

Other Possible Experiments

(i) *To compare the moments of two magnets.*—From Eq. (1),

$$(\mathrm{d}H/\mathrm{d}x)M_1h = bi,$$

where

$$\mathrm{d}H/\mathrm{d}x = -2M(3x^2+l^2)/(x^2-l^2)^3;$$

thus

$$\frac{2M(3x^2+l^2)}{(x^2-l^2)}M_1\frac{h}{b} = i. \qquad (6)$$

Then for magnets of moments M_2 and M_3 and lengths $2l_2$ and $2l_3$, each at distance x from the magnetometer of moment M_1,

$$\frac{2M_2(3x^2+l_2^2)}{(x^2-l_2^2)^3}\,\frac{(x^2-l_3^2)^3}{2M_3(3x^2+l_3^2)} = \frac{i_1}{i_2},$$

or

$$\frac{M_2}{M_3} = \frac{i_1}{i_2}\,\frac{(3x^2+l_3^2)}{(3x^2+l_2^2)}\,\frac{(x^2-l_2^2)^3}{(x-l_3^2)^3} = \frac{i_1}{i_2},$$

for short magnets or magnets of equal length.

(ii) *To find the moment of a magnet.*—It would first be necessary to find the constant of the magnetometer M_1h/b shown in Eq. (6).

If a flat coil of radius a and N turns carrying a current I amp were used, its axis being in line with the axis of the magnetometer magnets M_1 and its center at distance x from the center of the magnets M_1, then the field due to this coil would be

$$H = \frac{2\pi Na^2I}{10(a^2+x^2)^{3/2}};$$

thus

$$\frac{\mathrm{d}H}{\mathrm{d}x} = -\frac{6\pi NI}{10}\cdot\frac{a^2x}{(a^2+x^2)^{5/2}},$$

and since, from Eq. (1),

$$(\mathrm{d}H/\mathrm{d}x)M_1(h/b) = i,$$

the value of M_1h/b could be determined, since both i and $\mathrm{d}H/\mathrm{d}x$ are known.

Once the value of this constant was established it could be used in Eq. (6) to find the value of M.

[3] S. H. Ayliffe, *J. Inst. Elec. Eng.* **93**, Part 1, 509 (1946).

The next objective in science . . [is] . . . the teaching of science to the young, so that when the whole population grows up there would be a far more general background of common sense, based on a knowledge of the real meaning of the scientific method of discovering truth. —DAVID O. WOODBURY, *Beloved scientist*, a life of ELIHU THOMSON.

A Problem about Moving Charges

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National Chekiang University, China

IT is generally agreed that electromagnetic phenomena can be best explained by Maxwell's theory of the electromagnetic field, in which electric and magnetic actions are considered as propagated with finite velocity through a medium. In most problems, however, we are concerned with charges and magnets at rest or moving with steady small velocities. It happens that in such problems the assumption of an instantaneous "action at a distance" leads to the same results as an action propagated with finite velocity, and it is, moreover, a much simpler assumption to apply. As a result, many students fail, in their courses, to get a proper grasp of the role of the medium in Maxwell's theory.

The inadequacy of the theory of instantaneous actions can be most easily illustrated by problems involving swiftly moving charges. One of the simplest of these problems is discussed in this note.

Let two equal charges A and B, at a distance d apart, move with velocity v parallel to the x axis. From the point of view of an observer at rest, each charge will exert both an electrostatic and a magnetic force on the other. The magnetic force, computed on the basis of instantaneous action at a distance, will be

$$F=H(ev/c)=e^2v^2/d^2c^2. \qquad (1)$$

Here, e is the charge in esu and H is the magnetic field in emu at one charge due to the other, as computed by the law of Biot and Savart. From the point of view of an observer moving with the charges, the magnetic force will be zero. It follows that the resulting acceleration will be different for different observers moving at constant relative velocity. This is, of course, a paradoxical conclusion. If v is restricted to values small compared with c, the magnetic force will be negligible and the difficulty will not arise.

The difficulty is avoided entirely if the forces and accelerations are computed on the basis of action propagated at finite velocity. To show this, let us find the components of the electric and magnetic fields at A and B, first from the point of view of an observer moving with the charges, and then, with the aid of a Lorentz transformation, from the point of view of an observer at rest. The former will be

$$\begin{aligned}&E'(A)_x=E'(A)_z=0,\quad E'(A)_y=-e/d^2;\\ &\qquad H'(A)_x=H'(A)_y=H'(A)_z=0;\\ &E'(B)_x=E'(B)_z=0,\quad E'(B)_y=e/d^2;\\ &\qquad H'(B)_x=H'(B)_y=H'(B)_z=0.\end{aligned}$$

A Lorentz transformation applied to these yields for the components from the point of view of the observer at rest:

$$\begin{aligned}&E(A)_x=E(A)_z=0,\ E(A)_y=-(e/d^2)(1-\beta^2)^{-\frac{1}{2}};\\ &\qquad H(A)_x=H(A)_y=0,\\ &\qquad H(A)_z=-(\beta e/d^2)(1-\beta^2)^{-\frac{1}{2}};\\ &E(B)_x=E(B)_z=0,\ E(B)_y=(e/d^2)(1-\beta^2)^{-\frac{1}{2}};\\ &\qquad H(B)_x=H(B)_y=0,\\ &\qquad H(B)_z=-(\beta e/d^2)(1-\beta^2)^{-\frac{1}{2}},\end{aligned} \qquad (2)$$

in which $\beta=v/c$. Thus the components of the forces on A and B must be

$$\begin{aligned}&F(A)_x=F(A)_z=0,\ F(A)_y=eE(A)_y-e\beta H(A)_z\\ &\qquad =-(e^2/d^2)(1-\beta^2)^{\frac{1}{2}},\\ &F(B)_x=F(B)_z=0,\ F(B)_y=eE(B)_y-e\beta H(B)_z\\ &\qquad =(e^2/d^2)(1-\beta^2)^{\frac{1}{2}}.\end{aligned} \qquad (3)$$

The components of the accelerations of these charges are

$$\begin{aligned}&a(A)_x=a(A)_z=0,\ a(A)_y=F(A)_y(1/m)(1-\beta^2)^{\frac{1}{2}}\\ &\qquad =-(e^2/md^2)(1-\beta^2),\\ &a(B)_x=a(B)_z=0,\ a(B)_y=F(B)_y(1/m)(1-\beta^2)^{\frac{1}{2}}\\ &\qquad =-(e^2/md^2)(1-\beta^2),\end{aligned} \qquad (4)$$

where m is the rest mass of each charge, or the mass with respect to an observer moving with the charges, and $m(1-\beta^2)^{-\frac{1}{2}}$ is the relativistic mass.

From the point of view of an observer moving with the charges, however, the only forces acting on the charges are electrostatic. Their components are

$$\begin{aligned}&F'(A)_x=0,\ F'(A)_y=-e^2/d^2,\ F'(A)_z=0;\\ &F'(B)_x=0,\ F'(B)_y=e^2/d^2,\ F'(B)_z=0.\end{aligned} \qquad (3')$$

Therefore, the components of the accelerations

are

$$a'(A)_x=0,\ a'(A)_y=-e^2/md^2,\ a'(A)_z=0,$$
$$a'(B)_x=0,\ a'(B)_y=e^2/md^2,\ a'(B)_z=0. \qquad (4')$$

According to the special theory of relativity, if the acceleration of a particle with respect to one observer is a, and the acceleration of the same particle with respect to a second observer, in uniform motion with respect to the first, is a', and further, if the particle happens to be instantaneously at rest relative to the second observer, than a and a' are connected by the transformation

$$a_x=a_x'(1-w^2/c^2)^{3/2},\ a_y=a'_y(1-w^2/c^2),$$
$$a_z=a_z'(1-w^2/c^2). \qquad (5)$$

Here w is the relative velocity of the two observers.

A comparison of Eqs. (4) and (4′) shows that in them the accelerations a and a' are related in exactly the manner stated in Eq. (5). Thus the derivation of Eqs. (4) and (4′), based on relativistic electrodynamics, leads to a result consistent with relativistic mechanics and no paradox appears. The point to be emphasized is that it is only because relativistic electrodynamics is based on action propagated with finite velocity c that this agreement is possible.

The author acknowledges with thanks the considerable help he has received from Dr. William T. Payne in achieving a straightforward and clear presentation of this topic.

Photosensitive Glass

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PHOTOSENSITIVE glass is a new type of glass in which a permanent photographic image of exceptional fidelity is reproduced after exposure to ultraviolet light and development by heat. The undeveloped glass is clear, of high quality, and not distinguishable by its appearance from ordinary glass. There is no evidence of particles in the glass when examined under a microscope. The glass may be made in any desired shape consistent with ordinary commercial methods, for example, by drawing, rolling, blowing, or pressing. The chemical composition of the glass is covered by patents.

Two steps are required to impart a photographic image to the glass: first, exposure to ultraviolet light through an ordinary film or glass negative; second, development at high temperature. The light source must produce radiation in the 310–340 millimicron region of the ultraviolet. After exposure, the glass is still clear and transparent. The latent image is developed by simply heating the glass uniformly to approximately 565°C. Development may be completed at any time after exposure provided the glass is protected from ultraviolet. Times of exposure and development can be varied to control the depth of penetration and resultant color.[1]

The method of development of the image, the transmission properties of the glass, density and exposure time relations were studied. These will be considered in the following sections.

Development

A circular sample of the photosensitive glass was placed on a porcelain crucible, in a horizontal position, and developed in an electric furnace for 30 minutes at 565°C. Temperature measurements were made with a chromel-alumel thermocouple, with the hot junction 5 mm above the middle of the specimen. As a result of this treatment the sample was badly distorted. To overcome the difficulty the glass was placed on a carbon slab, raised about 3 cm above the floor of the furnace, and then developed.

The recommended development temperature of 630°C was found to be too high. At this

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[1] Anon., "Photosensitive Glass Developed by Corning," *Chem. and Eng. News* **25**, 1822 (1947).

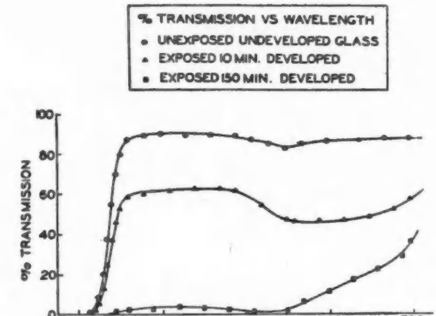

FIG. 1. Curves of wavelength *versus* percentage of transmission for unexposed and exposed glass.

temperature the glass became too soft to hold its shape.

Transmission Properties

The transmission properties of photosensitive glass were studied using tungsten lamp, mercury arc, carbon arc and hydrogen arc sources. Data for the transmission curves shown in Fig. 1 in the visible region from 400 to 700 millimicrons were obtained using a Cenco-Sheard spectrophotelometer. The associated galvanometer was mounted on a Julius suspension. The source was a simple projector with a 6-volt, 18-ampere tungsten lamp, power being received from a voltage-regulated power supply so that the illumination could be held constant during a series of readings. A converging lens, 7.6 cm in diameter, focused the light on the slit. The output of the photoelectric cell in the spectrophotelometer was connected through a shunt box to the galvanometer. Because of the predominance of yellow and red in the spectrum of the lamp, and the high sensitivity of the photoelectric cell in that region, it was necessary to reduce the voltage applied to the light source when using long wavelengths. Transmission characteristics of the glass in the ultraviolet region were determined by a Beckman spectrophotometer, with hydrogen arc source.

The transmission curves reproduced in Fig. 1 indicate that after development the glass exposed the longer time to ultraviolet light is not only more dense than the less exposed glass, but has an increasing transparency at the longer wavelengths as well as a gentle maximum in the 400-millimicron region. Two noticeable changes were found after exposing and developing. The glass which was exposed for a shorter time took on a bluish tinge. The glass exposed for a longer period became reddish. On closer examination by viewing edgewise it was found that the well exposed glass did show a blue layer, but this layer had penetrated farther into the glass. The blue layer was at the edge of the red coloration. As the exposure time was increased the thickness of the red layer in the glass increased. The thin blue layer was the same thickness in each case, but moved into the glass.

The thickness of the blue layer may be increased by exposing the glass to radiation of longer wavelength. For example, if an intense carbon arc is used and if the very short wavelengths (below 320 millimicrons) are filtered out, the red and blue colorations will still be present, but the thickness of the blue layer will be greater.[2]

A sample of the glass was exposed to a mercury arc-quartz spectrograph combination for 8 hr and then developed at 565°C for 1000 sec. Only one broad spectrum line was visible, and this was very faint. A longer exposure time of 23 hr was used to try to strengthen this line, and to bring out other lines. After development the same line was present, with greater intensity this time. No other lines were visible, as Fig. 2 shows. The line affecting the glass was in the 312–313 millimicron region

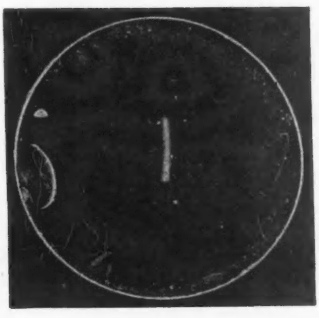

FIG. 2. Sample of photosensitive glass showing the 312–313 millimicron line of the mercury arc spectrum.

[2] R. H. Mariner, private correspondence from Corning Glass Works.

of the arc spectrum. The 312 and 313 millimicron lines could not be resolved with the instrument used.

The sensitivity curve of the glass has a peak at about 320 millimicrons.[2] On either side the 302- and the 334-millimicron lines are already too far away from the sensitivity peak to affect the glass. Since this spacing of strong lines is characteristic of the mercury arc, there can be no control of the blue layer thickness with this source. It is, however, evident that the region of very low transmission and the region of sensitivity of the glass coincide.

When a small carbon arc and quartz spectrograph were used, and the glass was exposed for 22 hr, no results were observed when the glass was developed.

The curves in Fig. 1 show that the percentage transmission for the three glass samples falls off rapidly at short wavelengths, beginning close to the 360-millimicron region. At 310 millimicrons transmission is barely perceptible even with the two least exposed glasses. The well exposed glass absorbs all wavelengths below 340 millimicrons.

An experiment was done with a hydrogen arc whose spectrum extended to shorter wavelengths than these limits. No light was transmitted through the glass in the range from 310 millimicrons to 200 millimicrons.

Influence of Development Time and Temperature

The photosensitive glass was exposed for 40 min to the mercury arc at a distance of 45 cm. This time was sufficient for satisfactory contrast, but not excessive, and was chosen to avoid any variations which might arise as a result of a reversal effect.

White light from the projector was sent through a No. 244 Corning filter and then through the photosensitive glass while it was developing in the furnace. The emergent light was received on a photovoltaic cell, permitting an observation of the change in transmission characteristics during developing. Figure 3 shows the results in graphical form.

There was no change in the transmission characteristics of the glass for the first 3500 sec, while the furnace was warming up. The glass

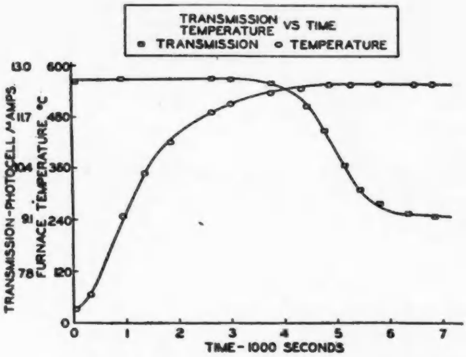

FIG. 3. Curves showing the relationship between transmission and development time, and between furnace temperature and development time.

showed no signs of developing until the temperature had reached 540°C. Since the prescribed development temperature was far higher than this, an arbitrary chosen constant furnace temperature of 565°C was maintained. The temperature was held constant until there was no further change in the transmission characteristic of the glass. The total time of development after the initial warming up period was 2750 sec. After development the temperature was lowered, but the transmission characteristic remained constant.

Variation of Density with Time of Exposure and Time of Development

Tests were made to see how different development times would change the density of variously exposed pieces of glass. The time of development was taken to be that time during which the temperature in the furnace was between 550 and 565°C. The glass samples were exposed at a distance of 45 cm from the mercury arc lamp for 5, 10, 20, 40, 80 and 160 min. After exposure the glass samples were placed in a special holder of brass tubing, separated by metal rings, and held secure by a large oversized ring. This made possible the simultaneous development of several samples under nearly identical conditions. The specimens were heated for a prearranged time, removed from the furnace and the density of each glass determined. The ends of the furnace were sealed to eliminate circulation of air during the development. The

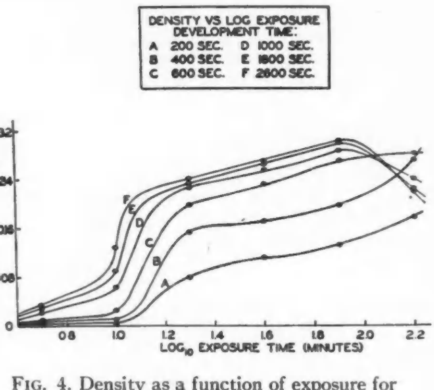

FIG. 4. Density as a function of exposure for several development times.

samples were then replaced in the tube and developed again. This was repeated five times, so that there were six development times for a total of 2600 sec.

The relation used to determine density was

$$D = \log_{10} I_0 / I,$$

where I_0 is the intensity of light transmitted through an unexposed piece of glass, and I the intensity of light transmitted through the exposed and developed glass. The intensity readings represented in Fig. 4 were made using a photovoltaic cell with a broad beam of emergent light covering the entire face of the cell.

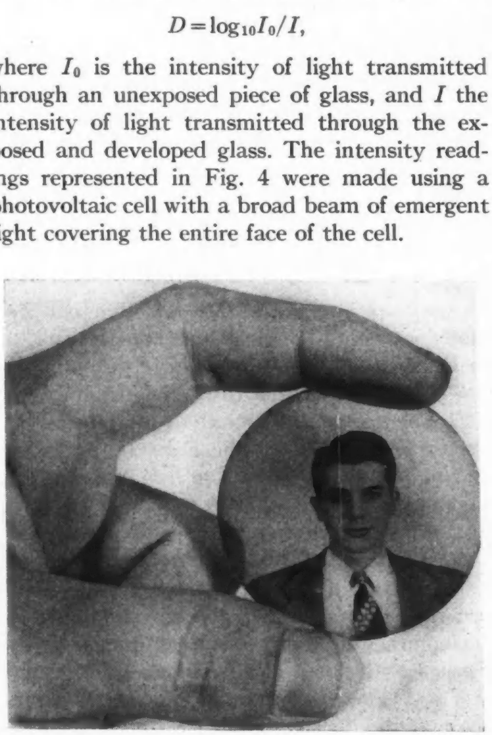

FIG. 5. Sample of photosensitive glass containing an image from a lantern slide negative.

One of the important relations characteristic of photosensitive materials is that between the time of exposure and the density. The normal curve for a photographic film may be found in standard text books on photography as a plot of density against $\log_{10} t$, where t is the exposure time.[3] The curve obtained with photosensitive glass may be compared with that of a normal photosensitive emulsion. Figure 4 gives curves characteristic of various stages of development. Curves A, B and C do not follow the normal curve, for the density continues to increase in the region of abnormal over-exposure instead of decreasing. Curve F represents the totally developed glass.

For a photographic emulsion a low value of density occurs upon development when the material has not been exposed. This is called fog. A sample of the photosensitive glass which had not been exposed was heated for 3000 sec at a temperature of 565–600°C. After cooling there were no signs of fog, since in this case the density remains zero after development. The underexposed region for the glass extends from about $t = 5$ to $t = 10$ min. This portion of the curve is similar to a normal curve in the under exposed region. The region of correct exposure is that portion of the curve between $t = 10$ and $t = 15$ min. On the logarithmic scale of Fig. 4 it appears rather narrow.

From $t = 15$ to $t = 80$ min is the region of overexposure. If compared with the curve of a normal photosensitive emulsion, it is seen that this region covers a much greater range on the exposure time coordinate than is the case with the normal photographic emulsion. This difference may be due to the fact that the exposed and developed glass has two layers of color, red and blue, as mentioned above. Initially, with small exposures, the curve is normal since only the blue color is present. However, with longer exposures the red color begins to come into the glass and the combination of these two colors gives rise to the broad region.

The region beyond $t = 80$ min is the abnormally overexposed region, where partially de-

[3] C. E. Kenneth Mees, *Theory of the photographic process* (Macmillan, 1944); Keith Henney and Beverly Dudley, *Handbook of photography* (Whittlesey House, 1939); J. E. Mack and M. J. Martin, *The photographic process* (McGraw-Hill, 1939).

veloped samples show an increase in density, and a totally developed sample, as shown by curve F, Fig. 4, shows a decrease in density as a result of very long exposures. This reversal effect is the same as that given by a normal photographic emulsion. The time of development was chosen so that there would be a marked change between successive curves in Fig. 4. Beyond 2000 sec the effects of further development were hardly noticeable.

The gradient γ, or slope of the curve in the correct exposure region, is an important defining characteristic of a photosensitive material.

$$\gamma = dD/d(\log_{10}t),$$

where dD is the incremental change of density with the corresponding incremental change of $\log_{10}t$. Values of γ for the glass samples are listed in Table I.

Photographic Prints

The mechanism of image production is not clearly understood. Apparently, submicroscopic metallic particles are present in solution in the glass mix. The incident ultraviolet light precipitates these particles, and they become colored. A three-dimensional effect is observed because the shadowed areas penetrate farther into the glass than the highlighted areas. Reproduction of extremely fine detail may be due to the lack of grain in the glass, and to the smallness of the particles making up the image. The metallic particles are much smaller than the silver grains in photographic emulsions.[1]

TABLE I. Values of γ for photosensitive glass.

Time of development (sec)	200	400	600	1000	1600	2600
Gamma	0.44	0.80	0.85	0.98	1.41	2.40

An ordinary Ansco photographic film negative was placed over the glass and the combination exposed for 10 hr to a mercury arc at a distance of 45 cm. When the glass was developed it was found that it had not been affected by this exposure. The film had absorbed all of the effective radiation. Other photographic films do not have this absorption property to nearly so great an extent.

It was well known that lantern slide glass plates transmit the ultraviolet light which affects the photosensitive glass. A lantern slide negative was, therefore, bound to the photosensitive glass and exposed for 15 hr. The glass was developed into a clear, distinct picture with a three dimensional effect, as shown in Fig. 5. The contrast in this reproduction is due to the red color of the exposed glass and to the use of orthochromatic film in copying it for publication. No filters were used.

The samples of photosensitive glass used in this investigation were obtained through the courtesy of the Corning Glass Works. Investigations of other optical properties of the glass are in progress.

One thing that never seems to occur to some of those who worry so loudly about the discretion of scientists is that the information in question, in many instances, was not given by the government to the scientist in the first place. Rather it was given by the scientist to his government. And as many of us know from personal experience it was sometimes most difficult to get the government to listen.

The history of the atomic bomb project is an interesting case in point. News of the discovery of uranium fission reached this country in January, 1939 from Germany. Within a short time quite a few American physicists recognized the possibility of useful release of atomic energy and of making an atomic bomb. Then started a process of trying to interest the government with no apparent action resulting. We physicists in the meantime voluntarily adopted secrecy policies which kept this information from the public and from other countries.

After some months of frustration a direct appeal to the President was made and he saw to it that a program of work was started under the general supervision of my predecessor, Dr. L. J. Briggs. The secret was so well kept that most of the staff of the National Bureau of Standards were unaware of the existence of an atomic bomb project before the official announcement was made.—E. U. CONDON (1948).

Ancient Science in the Modern Curriculum

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THE present quickened interest in physics as a cultural subject gives some teachers of physics not only an opportunity, but an obligation, to consider with their classes the nature and history of scientific activity. Much light is thrown on these matters by a consideration of Greek and Roman science. A very little reading in the subject is enough to convince one that Greek science was far from being the contemptible guesswork that it is often supposed to have been. By inquiring into the science of the classical period we can see things about science which might not be obvious from a study of the modern world alone, but which are of the utmost importance in the present period of uncertainty concerning the relation of science to society. The purpose of this article is to outline a few of the accomplishments of the Greeks, showing the extent to which physical science developed in their hands, and to point out some of the inferences that can be drawn from the rise and fall of science in the ancient world.

We may begin by observing that the Greeks invented science. At least until the advent of quantum mechanics, science has rested on the hypothesis of "a constant and universal sequence of cause and effect." The Greeks were the first to believe in this hypothesis. To be sure, not all Greeks believed in it, but not all Americans believe in it today, many of our religious practices being at variance with it. The important thing is that an appreciable number of Greeks believed in the hypothesis, while nobody in earlier cultures seems to have done so.

As far as method is concerned, the fundamental characteristics of physical science are (i) observation or experimentation and (ii) expression of findings in mathematical form. We take these tremendous intellectual inventions for granted now, but if the Greeks had bequeathed nothing else to our civilization, we should still owe them eternal respect for this legacy. One may ask, however, whether they did anything with science after they had invented the principles on which it rests, and whether we are not justified in regarding them as mere speculators rather than as scientists. Before summarizing a few of the actual accomplishments of the Greek scientists, it will be well to mention some of the reasons why Greek science seems meager to us.

The first and most important reason is that, as Sarton[1] puts it, science and technology are almost the only human activities which are *cumulative*. Failure to recognize this difference between science, in which the work of many generations is at our disposal, and the arts, in which each generation fends for itself, leads us to underestimate Greek accomplishment in science. Second, our impression of Greek science is too often based on the works of Plato and Aristotle, whose chief interests and skills lay in other fields of learning than physical science. Third, the literary tradition has had such a hold on classical scholars that they have almost completely ignored the Greek scientists. Finally, the works of the best Greek scientists were at variance with the religious views of the Christians, Jews and Moslems on whom we are dependent for the transmission of Greek learning. Hence our knowledge of these works is fragmentary.

Let us now consider some of what the ancient scientists actually accomplished, casting an occasional glance at the relationship between science and society during this period.[2] To review all of the scientific work of the Greeks is of course impossible and unnecessary for the present purpose. By considering only the progress and decline of the ancient world's knowledge of the solar system, including the earth, we shall see that the Greeks, using observation, experiment, and mathematics, gained a very considerable store of positive knowledge, that the Greek scientific enterprise succeeded to a degree that commands respect even in our time, and that it then withered away. By examining the causes of the

[1] G. Sarton, *Introduction to the history of science* (Carnegie Institution of Washington, 1927), vol. I, p. 4.

[2] The principal references used in preparing this summary have been Sarton (reference 1) and G. Singer, *A short history of science* (Oxford, 1941). Singer's excellent book can be recommended to anyone seeking an orientation in the subject.

withering, we may hope to throw light on problems of today.

Astronomical records, used for predicting the proper time to plant crops, existed in Mesopotamia at a very early date. The annual flooding of the Nile valley is said to have given rise to the science, or perhaps more properly the technology, of surveying; at any rate, "geometry" is Greek for "earth-measurement." These pre-Greek efforts yielded a considerable accumulation of recorded data and rules of thumb, but science as an effort to correlate phenomena by means of universally valid principles seems certainly to have been invented by the Greeks. In fact, its invention is usually attributed to a particular Greek, Thales of Miletus,[3] who flourished about 580 B.C. Thales discovered some geometrical generalizations, such as some of the properties of similar triangles. These properties he used to solve practical problems, for example, finding the distance from shore of a ship at sea.

Thales also thought about the nature of things, reaching the conclusion that everything is made of water. This is naive, but not silly; it was not long ago that we thought everything was made of hydrogen. The important point is that Thales tried to explain the universe in rational terms.

Pythagoras (530 B.C.)[4] usually considered the inventor of mathematical proofs, also founded mathematical physics with his discovery of the simple ratios of the lengths of strings sounding the musical intervals (when the strings all have the same tension and linear density). *This discovery must have been based on experiment.* With mathematical proofs, the idea of experimenting, and the idea of applying mathematics to the results of experiments, science in the modern sense was fairly started.

While the Pythagorean school was flowering in southern Italy, the Miletan school[5] founded by Thales was still in action. One of Thales' disciples, Anaximander (570 B.C.), used a sun dial to study the motions of the sun, and to determine the solstices and equinoxes. Another, Anaximenes (530 B.C.), reached the conclusion that the moon shines by reflecting light that comes from the sun.

About 450 B.C. wealthy Miletus, the greatest Greek city of its time, was conquered by the Persians, and the intellectual center of Greece shifted from Asia Minor to Athens. Anaxagoras, a contemporary and friend of Pericles and Euripedes in Athens, thought the earth to be flat, but correctly explained eclipses of the sun and the moon. For teaching that the sun is a huge ball of white-hot stone[6] he was charged with impiety and is said to have found it wise to return to Asia Minor.

Anaxagoras' contemporary, Socrates (430 B.C.), must be mentioned because of his two-fold influence on the progress of science, which he aided by combating loose reasoning, but impeded by turning men's attention to ethical problems, with which he himself was preoccupied.

Socrates' most distinguished pupil, Plato (390 B.C.), was also concerned largely with problems of conduct, but contributed much to the logical structure of mathematics and fostered the study of astronomy. His disciple, Eudoxus (370 B.C.), put astronomy for the first time on a mathematical basis, developing a very ingenious system of concentric spheres to account for the retrogressions in the motions of the planets. It is important to mention that he did not regard these spheres as material. He also measured the length of the solar year, making an error of only about one minute.

The idea that the earth is a sphere dates back to Pythagoras; it seems to have won general acceptance at least as early as the generation after Plato. Heraclides, another of Plato's pupils, postulated the rotation of the earth to account for the apparent diurnal rotation of the heavens, and discovered that Venus and Mercury—the two planets whose orbits are smaller than that of the earth—revolve about the sun.

Aristotle (340 B.C.), also a pupil of Plato,

[3] An ancient city on the west coast of what is now Turkey.

[4] The dates of important men are mentioned in order to establish a rough scale of time. They indicate the year when the man was about forty.

[5] The reader should perhaps be warned at the outset against one of the primary misconceptions about Greek science: that it was a sporadic activity, engaged in from time to time by men working in isolation. It will become clear that Greek science was actually a *continuous* activity, carried on in schools that endured for generations by men who were cognizant of the activities of other workers. The misconception doubtless arises from the random and fragmentary nature of the documents which happen to have survived the long period when they were regarded without interest or with disfavor.

[6] He seems to have based this judgment on a huge meteorite which fell near the Hellespont. See Sarton, reference 1, p. 86.

understood the function of research and contributed enormously to biology by careful investigation, but his work in physics was less fortunate. From our point of view, he made two major mistakes: he attempted a synthesis explaining all dynamical phenomena in terms of first causes, without having a sufficiently detailed knowledge of the phenomena, and he underestimated the importance of quantitative observation. His extensive writings on physics were very influential during the Revival of Learning; they contribute heavily to the Greeks' reputation as poor scientists.

After the almost coincident deaths of Aristotle and Alexander the Great, Alexandria took the place of Athens as the scientific center of the western world. More significant than the change in site was the accompanying change in method. Scientists became a specialized group, no longer seeking to embrace all knowledge and study the universe as a whole.

Perhaps the greatest scientist at Alexandria was Aristarchus of Samos (270 B.C.), who ranged through all of the mathematical sciences of his time, but is chiefly noted for his works on astronomy, of which the only survivor is *On the sizes and distances of the sun and the moon.*[7] The relative distances of the sun and the moon from the earth were determined by estimating the angle A between lines of sight to the sun and the moon when the moon was exactly at the half (Fig. 1). Aristarchus took A to be 87°, and computed the ratio of the desired distances. Since A is actually about 89° 50′, Aristarchus' result was

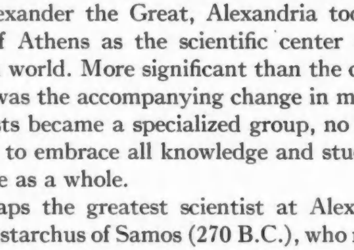

FIG. 1. Illustrating Aristarchus' method of finding the relative distances of the sun and the moon.

much too small—about 19 instead of 346. He knew from watching an eclipse that the sun and the moon subtend very nearly the same angle at the earth, and therefore was able to reason that the diameter of the sun must be about 19 times that of the moon, which made the volume of the sun about 7000 times that of the moon. Moreover, estimating the angle subtended by the moon at the earth to be[8] 2° and the diameter of the earth's shadow on the moon during a lunar eclipse to be twice the diameter of the moon, he deduced theorems to show the volumes of the moon, earth and sun to be as 0.05, 1 and 300. These figures are simplifications of those of Aristarchus, who, lacking trigonometric tables, calculated lower and upper bounds of the ratios.

Because of low accuracy in the measurements, these results are far from correct, but they are of great intellectual significance because they showed that while the moon is smaller than the earth, the sun is very much larger. It was perhaps this investigation that led Aristarchus to his crowning achievement, which was nothing less than the detailed anticipation of the Copernican hypothesis, in a book which is now lost. Several predecessors of Aristarchus had taught that the earth rotates on its axis, and Heraclides, seventy years or so earlier, had found that Mercury and Venus revolve about the sun. We are told by Archimedes, a younger contemporary, that Aristarchus took the sun to be one of the fixed stars, about which the earth and the other planets revolve in circular orbits; the moon revolves about the earth. He stated that the diameter of the earth's orbit is negligible in comparison with the distances to the stars. Therefore no parallax was to be expected, and the major objection to the motion of the earth (raised by Aristotle[9]) was removed. We know that Copernicus was aware of Aristarchus' system and mentioned it in a passage which he subsequently deleted.[10]

The successors of Aristarchus improved on his measurements of the sizes and distances of the moon and the sun, but his heliocentric theory was soon rejected, probably because it did not account accurately for the motions of the moon and

[7] For an English translation of this work, see Sir Thomas Heath, *Aristarchus of Samos* (Oxford, 1913), which also contains a summary of Greek astronomical progress before Aristarchus.

[8] In a presumably later work, Aristarchus announced that this angle is 30′, a much more accurate value.

[9] *De coels* ii, 14

[10] Heath, reference 7, p. 301.

the planets when improved observations, made with improved instruments, became available. As better observations brought to light more details of these motions, the astronomers "saved the phenomena," or made the theory fit the facts, by elaborating the number of epicycles in the geocentric system. Not until Kepler suggested elliptical orbits was the heliocentric theory a great improvement on the geocentric one. One is reminded of the rejection of Prout's hypothesis because of improved measurements of atomic weights, and its resurgence after the discovery of isotopes.

To go from Thales' idea of the earth floating on the water like a cork to Aristarchus' heliocentric theory took the Greeks just about three centuries. Among the astronomers who followed Aristarchus may be mentioned Hipparchus (150 B.C.), who is notable for his invention and improvement of astronomical instruments and his discoveries of a nova and the precession of the equinoxes, and Ptolemy (150 A.D.), whose chief accomplishment was the collection and exposition of Hellenistic knowledge of the heavens and the earth in his *Almagest* and *Geography*. The quality of Hipparchus' work may be judged from his value[11] of the mean lunar month, which differs from the present value by less than 1 sec. Ptolemy's works have been passed on to us by the Arabs, but were not translated into Latin until the close of the Middle Ages. Hipparchus worked partly, and Ptolemy wholly, at Alexandria; some of Ptolemy's work was original, but he depended very largely on his predecessors, particularly Hipparchus. In spite of living three centuries apart, these two men were almost collaborators. We therefore notice a striking decrease in the rate of progress, in comparison with the earlier period.

It is instructive to consider also the progress of knowledge concerning the earth itself. The work of Eratosthenes (235 B.C., a generation after Aristarchus) lay primarily in this field. His measurement of the diameter of the earth is an example of Greek science at its best. At Syene (Assuan) in Egypt, at noon on the summer solstice, a vertical post cast no shadow, indicating that the sun was directly overhead. Eratosthenes measured the length of the shadow cast at the

[11] *Cambridge ancient history* (Cambridge, 1928), vol. 7, p. 311.

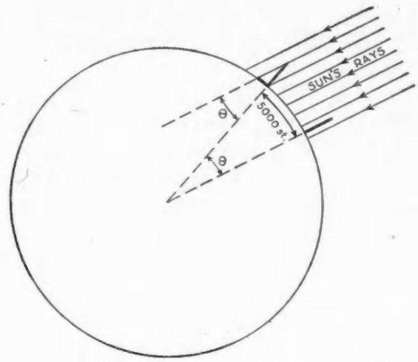

FIG. 2. Illustrating Eratosthenes' method of finding the diameter of the earth.

same time by a vertical rod at Alexandria, which he took to be 5000 stadia north of Syene. The lengths of the rod and its shadow gave the angle θ (Fig. 2) between the earth's radii to the two points, thereby enabling him to compute the diameter of the earth.[12] His measurement of θ was very good, but his value for the base line was 1 percent too small, and his value of 7820 statute miles[13] for the diameter of the earth is therefore 1 percent smaller than the true distance from pole to pole.

Eratosthenes also made a map of the world as he knew it, and suggested that only the width of the ocean prevented the circumnavigation of the globe. He estimated the breadth of Europe and Asia to be 7600 miles; actually the greatest distance from the European coast to the Asian coast, along a parallel of latitude, is about 7640 miles (Lisbon-Fusan).[14] He took Ceylon and the Shetland Islands to be the southern and northern limits of the inhabited world, and placed the Shetlands 3700 miles farther north than Ceylon. The error in this distance cannot be computed without knowing which part of the Shetlands he

[12] For the practical details of this experiment, see Pauly-Wissowa, *Real-Encyclopadie der Classischen Altertumswissenschaft* (Stuttgart, 1909), vol. 6, pp. 363–366.

[13] The length of his stadium is not known with certainty, but is thought to be 0.0977 statute miles (see Heath, reference 7, p. 339). The startling accuracy of *all three* of the measurements quoted here gives some confidence in the correctness of this conversion factor.

[14] It is possible, but unlikely, that Eratosthenes had in mind the longest great-circle arc between the coasts. The true value for this distance is 7800 miles (Lisbon-Singapore). In any case, if Columbus had known this distance and the diameter of the globe as well as Eratosthenes did, he would scarcely have set out to cross the ocean.

referred to, but the southern limit of Ceylon is 3750 miles south of the center of Mainland, the chief Shetland isle.

Four centuries later, Ptolemy's *Geography* showed some improvement in geographical knowledge, but here too the rate of progress had obviously fallen off. We may say that Ptolemy's map of the world, reconstructed from his latitudes and longitudes, represents correctly the broad features of northern Africa, southern Asia as far as Indo-China, and Europe exclusive of Scandinavia. In the roughly contemporary Roman world, however, Tacitus, writing on geographical matters, described the earth as flat, and by 600 A.D., when Isidore of Seville compiled his encyclopedia, the map of the world as far as Europeans were concerned had degenerated so much as to be scarcely recognizable.

We see, therefore, that from 550 to about 150 B.C., the Greek world developed to a high degree the technic of scientific investigation of certain types of problems. We have mentioned cosmography and geography; very considerable progress was also made in some branches of biological science, where dissection of animals and men led to anatomical discoveries which were in some cases not repeated until the nineteenth century. We naturally inquire why this scientific activity died out. The complete answer is of course very complex, but a few of the reasons may be mentioned.

When the empire of Alexander fell apart after his death, the unsettled political conditions were unfavorable to the development of science. The lot of most people was so miserable that philosophy concerned itself primarily with questions of ethics and religion; science seemed unimportant. In Egypt, however, stability and wealth permitted the ruling Greeks to foster the development of science by the foundation and support of an institute like a university, the Museum at Alexandria. Scholars and scientists came here from all over the Greek world, and ancient science entered on its golden age. These scientists were specialists, not so specialized as the modern scientist, but much more so than Plato and Aristotle. In the second century B.C. Rome established control over the Eastern Mediterranean, although Egypt retained nominal independence until about 50 B.C. The educated Romans tried to assimilate Greek culture, and succeeded to some extent in philosophy, literature and the fine arts, but they failed completely in science. Among the reasons for this were undoubtedly the following.

(i) Science failed to appeal strongly to them because of their religious convictions, which emphasized the importance of conduct and minimized the importance of understanding the material world. Partly, perhaps largely, for military reasons, they did have a strong interest in technology, but this of course is not science. The validity of the distinction is confirmed by the difference in results.

(ii) They never mastered the higher branches of Greek mathematics, and therefore could not produce original work in the exact sciences. This is probably a consequence of (i).

(iii) Above all, they failed to recognize that science is an *operation*, rather than a collection of facts. They tried to take over and preserve the *results* of Greek science. This attempt failed because the body of real knowledge inevitably became more garbled and diluted with each generation, so that it was almost entirely lost long before the fall of Rome.

The fact that science is an operation, rather than a collection of facts, is probably the most important thing the teacher of physics as a cultural subject has to impress on his present classes, if their preconceived notions of science are to be replaced by real understanding. Starting the elementary physics course with a lecture or two on ancient science affords a convincing demonstration, based on simple subject matter, that science is not a thing that inevitably progresses. It does not grow like a weed. On the contrary, it is a delicate organism depending in a very sensitive way on its environment. It has died before, and without the proper conditions of life it will die again.

NOTES AND DISCUSSION

Principal Series of Sodium in Absorption

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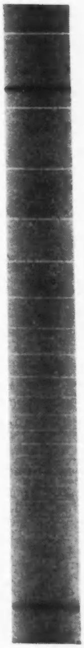
THIS note describes the results of a qualitative investigation into the principal series absorption in sodium vapor, using an absorption tube designed primarily for use by students in the third-year course in physics at the Massachusetts Institute of Technology.

The apparatus is described elsewhere,[1] but essentially consists of a ten-inch seamless stainless steel tube, surrounded by a heater coil and insulating material, and having water-cooled ends to which quartz windows are attached. Stockbarger's article describes the tube's possibilities as follows.

> Apparently the number of principal series members of sodium which can be observed with the apparatus is determined largely by the resolution of the optical system. With a mercury arc source, students obtain about 15 members with a small quartz spectrograph and 30 or more with a medium size instrument, · · · .

In the present work clean sodium metal was placed in the apparatus which was then outgassed at a high temperature. A 35-foot grating spectrograph, having a dispersion of 0.76A/mm in the first order and a resolving power exceeding 100,000, was used to obtain absorption by the sodium at temperatures above 600°C out of the high intensity continuum in the spectrum of a quartz pool-type mercury arc. The series from $3S-15P$(2444.195A) to the limit at about 2410A is reproduced in Fig. 1. The last series member detected on the original plates was the $3S-52P$ transition, which indicates a resolving limit of approximately 0.095A. Resonance broadening and broadening due to residual gases are probably the factors that limit the number of series members attainable.



–2444.195A

–2410A

FIG. 1. High members of the principal series of sodium in absorption.

The Stockbarger absorption tube is thus capable of resolving the principal series in sodium to a degree which compares very favorably with the early work of Wood.[2] Until very recently,[3,4] Wood's observation of the $3S-59P$ electron transition involved the highest quantum state ever observed. It is of interest to point out that this was observed with a 2.8-meter absorption tube, and that the limit of observation was probably set by the resolving power of his spectrograph since the last twelve lines were observed in an interval of less than one angstrom.

* Now at Carbide and Carbon Chemical Corporation, Isotopes Branch, Oak Ridge, Tennessee.
[1] D. C. Stockbarger, *J. Opt. Soc. Am.* **30**, 362 (1940).
[2] R. W. Wood, *Astrophys. J.* **43**, 73 (1916).
[3] H. R. Kratz and J. E. Mack, *Rev. Mod. Physics* **14**, 104 (1942).
[4] J. R. McNally, Jr., J. P. Molnar, W. J. Hitchcock and N. F. Oliver, to be published in *J. Opt. Soc. Am.*

On the Area-Moment Propositions of Mechanics

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THE proofs of the area-moment propositions in mechanics can be simplified over those usually given in the literature, and the propositions themselves can be put in a form of more general application, as follows.

Proof of the first area-moment proposition.—The differential equation of the elastic curve is

$$d^2y/dx^2 = M/EI, \tag{1}$$

where M is the bending moment, I the moment of inertia of the section, and E the modulus of elasticity. Integrating between the limits $x=a$ and $x=b$, we have

$$\int_a^b \frac{d^2y}{dx^2}dx = \int_a^b \frac{M}{EI}dx;$$

whence, for beams of constant cross section,

$$\left(\frac{dy}{dx}\right)_{x=b} - \left(\frac{dy}{dx}\right)_{x=a} = \frac{A}{EI}.$$

This is equivalent to

$$\tan\theta_b - \tan\theta_a = A/EI.$$

Finally, in view of the fact that the angles involved are very small, the last equation can be written in the form

$$\theta_b - \theta_a = A/EI, \tag{2}$$

where, as usual, A is the area between the limits $x=a$ and $x=b$ under the corresponding moment curve.

Proof of the second area-moment proposition.—Multiplying both sides of Eq. (1) by $(b-x)$, and integrating between the same limits as above, we have

$$\int_a^b (b-x)\frac{d^2y}{dx^2}dx = \int_a^b \frac{M(b-x)}{EI}dx.$$

Integrating the left-hand member by parts gives

$$y_b - y_a - (b-a)\tan\theta_a = Ab^*/EI,$$

where b^* is the distance of the centroid of the area from the line $x=b$. Rearranging and taking account of the fact that θ_a is small yields, finally,

$$y_b = y_a + (b-a)\theta_a + Ab^*/EI. \tag{3}$$

The last equation reduces to the usual form provided $y_a=0$ and $\theta_a=0$. The geometric interpretation of the three terms on the right-hand side is obvious.

It will be noted that the generalized form of the second proposition allows the deflection to be obtained in every case by purely analytic methods, without resorting to additional geometric constructions. In particular, by the application of both propositions, a sufficient number of algebraic equations can be written (by suitable choice of pairs of points a, b) to solve any problem completely.

Application of the generalized area-moment propositions.—Consider the case of any simply supported beam. In order to deduce the deflection y at a section located a distance x from the left-hand support ($x=0$), we can apply the second proposition twice, once setting $a=0$, $b=x$, and again setting $a=0$, $b=l$; whence we have

$$y=0+x\theta_0+A_{0x}b_{0x}^*/EI,$$
$$0=0+l\theta_0+A_{0l}b_{0l}^*/EI.$$

Elimination of θ_0 gives the deflection y directly.

Alternate form of the second area-moment proposition.—If we multiply both sides of Eq. (1) by $(x-a)$ and carry out the integration as before, there results

$$y_b=y_a+(b-a)\theta_b+Aa^*/EI.$$

In view of Eq. (2), this can be written as

$$y_b=y_a+(b-a)(A/EI+\theta_a)+Aa^*/EI,$$

where a^* now represents the distance of the centroid of the area from the *left-hand* vertical, that is, from the line $x=a$.

A Dynamic Atom Model

John B. Underwood
High School, Grass Valley, California

THIS Bohr atom model is comparable to a lecture table planetarium, in that atomic particles are made to describe circular orbits like planets. Obviously no atomic model can duplicate proportional dimensions. This is not even attempted in an astronomical planetarium. The purpose of this model is to demonstrate the effect of particle motion and to show how it may determine the properties of an atom. The nucleus is made up of particles representing protons and neutrons, caused to move about in the smallest space mechanically possible. When in motion, the model gives the impression of a single nuclear mass. About the nucleus revolve the planetary electrons in appropriate orbits, giving the impression of cloud layers rather than particles. Other interesting phenomena such as nuclear magnetic effects can be illustrated. But it is our purpose here merely to describe the mechanical action of the model.

The oxygen atom proved ideal for our model. The eight protons and the eight neutrons in the nucleus just fill the four spokes on each of the four spindles used. The electrons are secured at the ends of the wire spokes common to the protons, thus illustrating the electrostatic bond of attraction between them. The two K orbit electron spokes are shorter than those carrying the outer orbit electrons, illustrating energy levels.

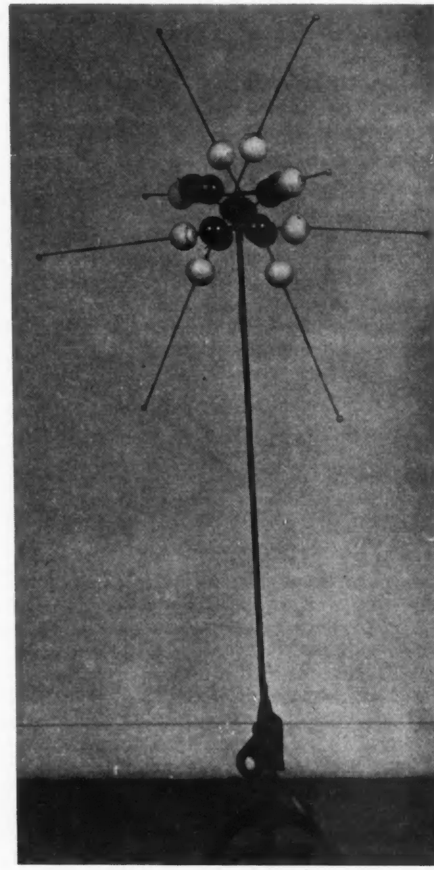

Fig. 1. The oxygen atom model at rest.

The general appearance of the model at rest is shown in Fig. 1. A vertical pipe, about four feet long and mounted on a tripod base, serves as a supporting stand and drive shaft housing. At the top of the pipe is the driving head upon which are mounted four spindles rotated by miter gears, one of which is driven by a shaft coming up through the supporting pipe from the driving gear operated by a crank at the base. Attached to each of the four spindles are wire spokes that carry the neutrons, protons and electrons. Hollow rubber balls are used for neutrons and protons, and spherical metal buttons for electrons. The white balls represent protons and the dark ones neutrons. These are free to rotate (see Fig. 2) on the wire spoke axis, thus illustrating magnetic effects within the nucleus. The eight wire spokes not carrying electrons terminate with the neutrons.

The mechanical details of the head are shown in Fig. 3. Standard $\frac{1}{4}$-inch pipe, a cross tee, three short nipples, a

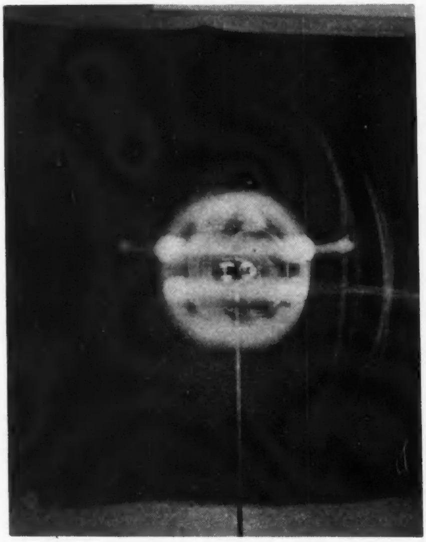

FIG. 2. The oxygen atom model in motion.

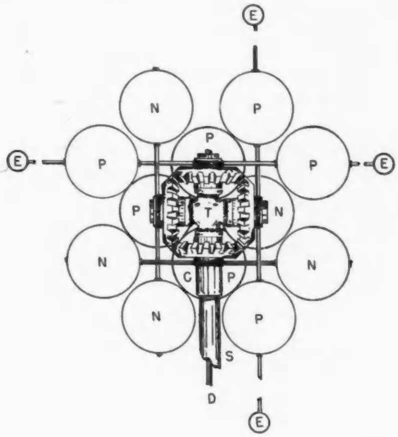

FIG. 3. Details of the driving mechanism of the oxygen atom model.

coupling, and two pipe caps were used for supports as shown. It will be seen that the miter gear at the top is the driver and that the other gears rotate in unison with it. The miter gears were cut from sheet metal and soldered on short brass tubing hubs which were made to fit the pipe nipples.

Figure 2 shows the appearance of the atom head when the crank at the base is turned two times per second. The blur at this speed is mostly photographic, the nuclear particles appearing to the eye rather to mill around as we would like to show them. At about double this speed the blur becomes real to the observer. The electrons appear only as circular orbits even at slow speeds. One of the horizontal orbits in Fig. 2 shows the paths of the two inner electrons. The remaining six orbits represent the outer electron orbits. In operation the electron orbits stand out much more clearly than it is possible to photograph, and the apparatus never fails to fascinate spectators.

RECENT MEETINGS

Southern California Section

The Southern California Section of the American Association of Physics Teachers presented its fourth annual competitive physics test for high school students on Saturday, May 15, 1948. Examination centers for this test were established at the University of California at Los Angeles, the University of California at Santa Barbara, the University of Redlands, and San Diego State College. The examination was two hours in length and included definitions of basic quantities and principles, plus problem work in the fields of mechanics, properties of matter, heat, sound, electricity, and light. The problems were graded on the basis of method as well as on accuracy.

Students participating were nominated for the test by their high school physics teachers. There was no limit to the number of students any one teacher could nominate. This year 193 students representing 47 high schools in the Southern California area competed. The fifteen top ranking students on the test were presented with certificates of award from the Section. In addition, duplicate certificates were presented to the schools represented by the winning students. These fifteen winners were:

Jon Mathews, Flintridge Prep. School, Pasadena; Francis Scott, John Marshall High, Los Angeles; Bill Colbert, Mark Keppel High, Alhambra; Alan Johnston, Van Nuys High, Van Nuys; Harry Watkins, Inglewood High, Inglewood; Bennie Ford, Venice High, Venice; Robert Lawrence, Wilson High, Los Angeles; David Johnston, Mark Keppel High, Alhambra; Lowell Doherty, Herbert Hoover High, San Diego; Bob Beavers, El Monte High, El Monte; Melvin Katz, Beverly Hills High, Beverly Hills; Phil Orville, Santa Monica High, Santa Monica; Gerald Parker, Venice High, Venice; Milo Webber, Pomona High, Pomona; and George Cooke, Hollywood High, Hollywood.

Grades on the test ranged from a high of 85.4 to a low of 11.7, with a satisfactory distribution between these limits. A particularly good spread resulting in a strong "cream separator" action was obtained at the top of the list.

In addition to the certificates of award, seven one-year college scholarships were available to the winners from the following schools:

California Institute of Technology, University of California (any campus), Occidental College, Pomona College, University of Redlands, University of Southern California, and Whittier College.

The winning students selected their scholarships from this list on the basis of rank and choice.

The test this year was the best of those given so far, and represented a great deal of work on the part of the high school test committee. The committee consisted of:

Professor Vernon L. Bollman, Occidental College, *Chairman*, Rev. Charles R. Coony, Loyola College, Professor Willard Geer, U.S.C., Professor Robert E. Holzer, U.C.L.A., Mr. Roy W. McHenry, Santa Monica City College, and Professor Kenneth Hurd, U.C.L.A. (member from host institution).

The annual competitive physics test has become one of the Section's most successful activities. The number of students and high schools participating has shown a very satisfactory increase from year to year, so that by now this annual test is regarded by both high school physics teachers and their students as one of the important events of the year. The support of the colleges of this region, as evidenced by the scholarships offered in behalf of the Association, indicates the importance with which these institutions regard the test.

FOSTER STRONG,
Secretary-Treasurer

Oregon Section

The eighteenth annual meeting of the Oregon Section of the American Association of Physics Teachers was held at the University of Washington on May 22, 1948. Seventy-two members and guests were present. Raymond T. Ellickson, President of the Section, presided. Fourteen papers were presented at the meeting.

At the business meeting the following officers were elected by unanimous vote: *President*, Walter P. Dyke, Linnfield College; *Secretary-Treasurer*, Fred W. Decker, Oregon State College; *Historian*, Brother Godfrey Vassallo, University of Portland. The fall meeting will be held at Vanport College on November 13, 1948.

1. **High-vacuum technique—an undergraduate study.** HARVEY E. WEGNER, *College of Puget Sound.*—A high-vacuum system using a mercury diffusion pump and a gas injection system for argon, helium, and neon has been constructed as an undergraduate project during the senior year. A few of the variety of tubes constructed for classroom demonstration are described, and some possible demonstrations for beginning physics students are discussed.

2. **Philatelic physics.** ALFRED B. BUTLER, *Washington State College.*—At least 26 stamps have been issued bearing the portraits of physicists, and many more celebrate physical discoveries or applications. If the stamps are mounted individually with accompanying biographical sketch or description and are displayed in a hall showcase, a surprising number of students will be attracted and will linger to read the accompanying material.

3. **Factors determining teaching effectiveness in the elementary laboratory.** T. W. LASHOF AND M. E. HOEHNE, *Reed College.*—A preliminary report was given on a project for the compilation of a compendium of laboratory exercises. The report covered the first step: the establishment of a set of questions for the evaluation of exercises to be included in the compendium.

4. **The setting up of the mirror and lens equations and their unambiguous interpretation.** A. E. HENNINGS, *University of British Columbia.*—The wave front method is used to show the relations between p, q, and f for mirror or lens. Real, inverted images are formed only when the wave front leaving the mirror or lens is converging. All other images are erect and virtual. Curves showing the relations between the object and image positions may be readily plotted.

5. **Microwave spectroscopy.** A. VAN DER ZIEL, *University of British Columbia.*—A survey of the techniques which are used in the investigation of rotational absorption spectra in the microwave region. The frequency of a klystron is swept linearly over about 10 Mc, and the signal generated by the klystron is passed through a wave-guide absorption cell and detected by a crystal diode. The detected signal is amplified and applied to the vertical deflection plates of a cathode-ray oscillograph. On the horizontal plates a sweep voltage is applied which is synchronized with the frequency sweep of the klystron. In this way absorption spectra are displayed on the oscillograph screen. A microwave spectroscope built along the above principles is nearing completion at the University of British Columbia.

6. **Infra-red absorption spectra.** A. M. CROOKES, *University of British Columbia.*

7. **Inertia effects in infra-red phosphors.** RAYMOND T. ELLICKSON, *Reed College.*—The visible light from an infra-red sensitive phosphor attains its maximum value after a period of the order of 0.001 second subsequent to the irradiation. An apparatus has been set up to measure this time of buildup. For one group of phosphors the time is independent of the intensity of the infra-red. For another group the build-up time is over one minute for very weak infra-red.

8. **Beta-ray energy measurements and disintegration schemes.** K. C. MANN, *University of British Columbia.*—A discussion of some of the methods in the field of beta-ray spectroscopy to determine the energies of beta- and gamma-radiations from radioactive nuclei is given. This is extended to the problem of determining the order of appearance of these radiations in the decay scheme. Some experimental results are quoted on Ra and Sb^{124}.

9. **Beta-decay at low energies.** EUGENE P. COOPER, *University of Oregon.*—Indications of the failure of the Fermi theory of beta-decay are found in the very "rapid" decay of H^3, the "abnormal" linearity with kinetic energy of the Cu^{64} electrons at energies below 100 kev, and the apparent superabundance of Cu^{64} positrons and Cd^{107} positrons in this low-energy region. It is suggested that radiation in the form of a continuous x-ray spectrum may accompany beta-decay in such a way as to be relatively most important at low energies where it could effectively modify the shape of the beta-spectrum. Such radiation may arise by virtue of "bremsstrahlung," since the elec-

tron is decelerated in the field of the nucleus which emits it. A semiclassical estimate of this effect is given, and a method for obtaining a quantum-mechanical estimate is outlined.

10. **The humanities in the physics curriculum.** RICHARD D. MURPHY, C.S.C., *University of Portland.*—The trend to general education and the possible doubling of college enrollment by federal aid demand a broadening of the outlook of the physics teacher and the physics student. The need for the required integration will have to be met by a philosophy of life, which, in our present state, will have to be culled by the individual from the non-technical part of the curriculum. The faculty will have to bear the burden of fostering interests in the social problems of the day both by its counseling and its attitudes.

11. **Humanities and other nontechnical subjects in technical curricula.** W. WENIGER, *Oregon State College.*—Attention is called to widespread activity in revising curricula with the purpose of offering students a more liberal education as well as preparing them for opportunities in the newer fields of physics. The principal aim is to keep alive an interest in all things, so that the physicists' methods of attack may be applied in other fields.

12. **Cerenkov radiation in the vicinity of the threshold.** JOHN M. HARDING and J. E. HENDERSON, *University of Washington.*

13. **Recent developments in the theory of spark discharge.** RONALD GEBALLE, *University of Washington.*

14. **Scintillation counters.** PHILIP A. GOLDBERG, *University of Oregon.*—A variety of problems associated with scintillation counter design, including such matters as phosphor selection, phosphor screen construction, photomultiplier spectral response, signal-to-noise ratio, and a special light shield for the photo-multiplier tube are treated. Various optical accessories have been used between the phosphor screen and the photo-multiplier tube to improve the energy differentiation characteristics of the counter. To increase the sensitive area of the counter, mirrors and a bank of photo-multiplier tubes have been used.

WILLIAM R. VARNER,
Secretary

ANNOUNCEMENTS AND NEWS

Meteorological Station at Brookhaven National Laboratory

Two meteorological towers for weather observations in connection with nuclear research are being completed at Brookhaven National Laboratory. The taller tower, to be 420 ft high, comparable in height to a 35- or 40-story building, will be the tallest structure on Long Island. The second tower, already complete, is 160 ft high. At five levels on the small tower, and at eight levels on the tall tower, will be platforms where observers may take readings from meteorological instruments.

Plans and design of the towers were supervised by the Brookhaven Meteorology Group, headed by Norman R. Beers, formerly Associate Professor of Aerological Engineering at the Naval Post Graduate School, Annapolis, Maryland. Working with the group is a special station of the United States Weather Bureau, headed by Mr. Raymond C. Wanta.

One of the chief purposes of the meteorological towers is to furnish information on wind currents in connection with safe operation of the Brookhaven nuclear reactor to be finished this fall. The nuclear reactor will be cooled by air. Large fans will conduct the air away from the pile, through an air duct, and up a 300-ft air stack on a hill near the reactor. In the air will be minute quantities of radioactive argon, a chemically inert gas, which will be dissipated harmlessly into the upper air. Study of wind velocity and direction, and atmospheric pressure, during operation will permit control of the pile so that air emanating from the stack will not settle on or near the ground.

Instruments on the weather tower and elsewhere on and near the Laboratory site will give precise information on wind currents. Operation of the reactor will be controlled at a low level or stopped entirely if it appears that radiation from the cooling stack under unfavorable conditions would not be completely harmless. With complete information on wind currents at upper and lower levels, and a knowledge of weather conditions on Long Island, scientists are thus assured that operation of the reactor will be harmless to neighboring communities, and to plant and marine life in the vicinity.

Another feature of the tall weather tower will be a smoke stack to carry smoke specially created for the purpose of studying wind and weather. This is a 20-in. steel pipe designed to dissipate smoke produced by an army surplus M-1 smoke generator, similar to those used by naval vessels and beachhead troops in laying down smoke screens to conceal an operation. It produces a harmless oil fog smoke, 10,000 ft^3 per min, and at some time during tests it may create near the Laboratory a small cloud visible for several miles.

Weather instruments on the towers will be mounted on beams that can be swung away from the towers, and then pulled in for observations. In addition, the observations will be recorded electrically on an instrument panel in a building 900 ft away. The building has to be a considerable distance from the towers so that eddies created by winds passing over the building will not disturb observations made by instruments on the towers. Two electrified cables between the towers will also carry instruments to record temperatures and wind differences between them.

The meteorology group at the Laboratory will afford facilities for study and research by scientists of the permanent staff and by others who come on leave from their regular jobs elsewhere. The Weather Bureau will also be orienting and training U. S. Weather Bureau personnel, who will later be assigned to similar work elsewhere.

New Members of the Association

The following persons have been made *members* or *junior members* (J) of the American Association of Physics Teachers since the publication, *Am. J. Physics* **16**, 369 (1948), of the preceding list.

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